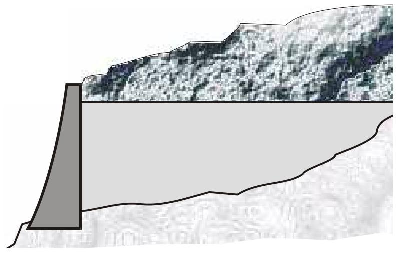
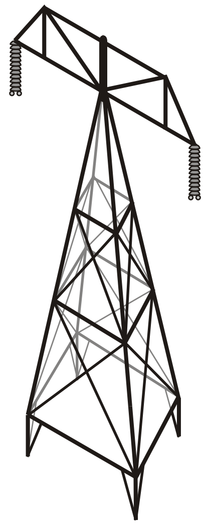
STRUCTURES

There are three main types of structure - mass, framed and shells.

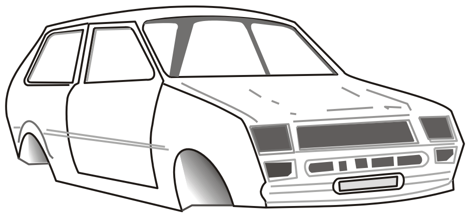
Mass structures perform due to their own weight.
An example would be a dam.



Frame structures resist loads due to the arrangement of its members. A house roof truss can support a load many times its own weight. Electricity pylons are good examples of frame structures.



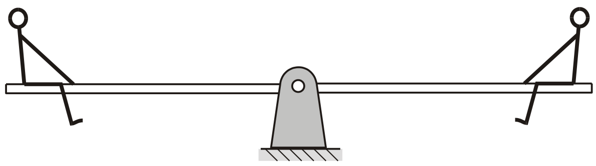
Shells are structures where its strength comes from the formation of sheets to give strength. A car body is an example of a shell structure.



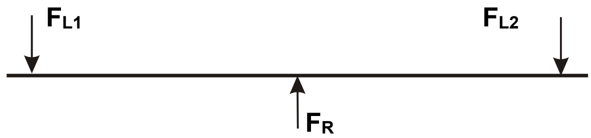
It is very important with any structure that we can calculate the forces acting within it so that a safe structure can be designed. Early structures were found to be successful due to the fact that they stayed up and many early structures are still with us, but many are not. The science of structures has been progressively improving over the centuries and it is now possible to predict structures behaviour by analysis and calculation. Errors can still be made, sometimes with catastrophic results.

FORCES

In the structure below, three forces are acting on it.



The diagram shown below can represent the forces in the above diagram.



F_{L1} and **F_{L2}** represent the force exerted due to the mass of the people. **F_R** is the reaction force. This type of diagram is known as a **free-body diagram.**

The reaction load **F_R** is found by adding the two downward forces together.

In any force system the sum of the vertical forces must be equal to zero.

$$\Sigma F_V = 0$$

$$F_R = F_{L1} + F_{L2}$$

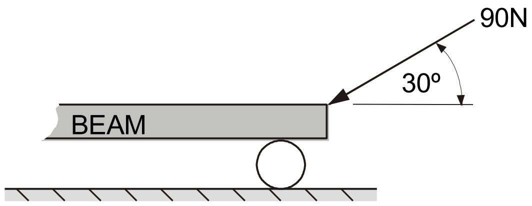
$$F_R = 810N + 740N$$

$$F_R = 1550N$$

RESOLUTION OF A FORCE

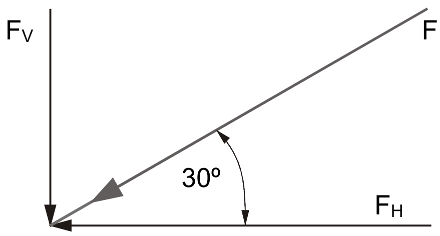
In some cases the loads may not be acting in the same direction, and cannot therefore be added together directly.

In the situation shown below the force is acting down at an angle.



This force can be split into two separate components:

A vertical component $\mathbf{F_V}$.
A horizontal component $\mathbf{F_H}$.



To resolve a force into its components you will have to know two things, its magnitude and direction.

Trigonometry is used to resolve forces.

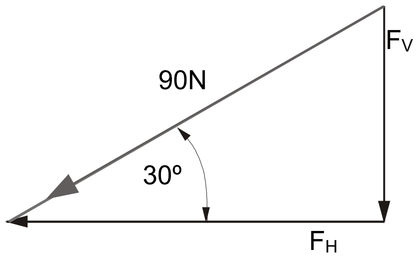
$$\cos = \frac{d}{}$$

Where - Hypotenuse = force, F

Opposite = vertical component, F_V

Adjacent = horizontal component, F_H

The diagram above can be redrawn as below.



To find the horizontal force, F_H

$$\cos 30° = \frac{F_H}{90}$$
$$F_H = 90 \times \cos 30°$$
$$= 77.94N$$

Horizontal force = 77.94 N

To find the vertical force, F_V

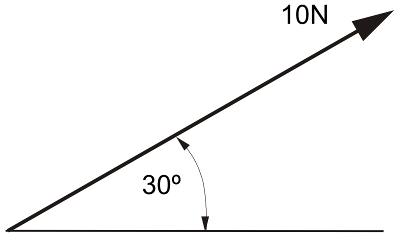
$$\sin 30° = \frac{F_V}{90}$$
$$F_V = 90 \times \sin 30°$$
$$= 45N$$

Vertical force = 45 N

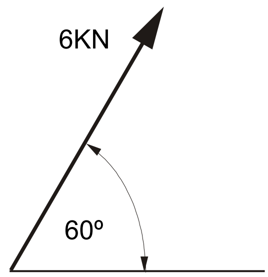
Task - Resolution of Forces

1. Resolve the following forces into their horizontal and vertical components.

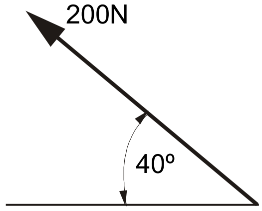
a)



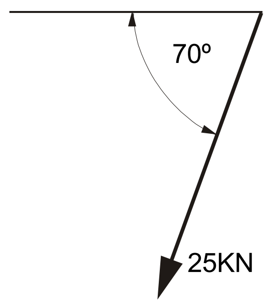
b)



c)

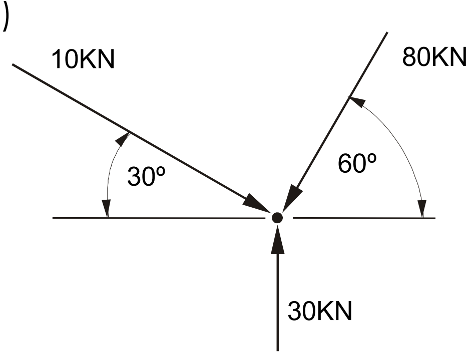


d)

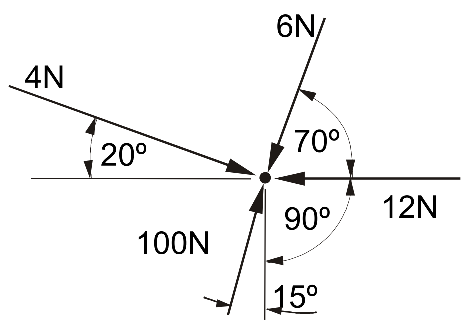


2. Find the resultant for these force systems. Find the horizontal and vertical components of each - add them up to find the overall component forces and then find the resultant.

a)

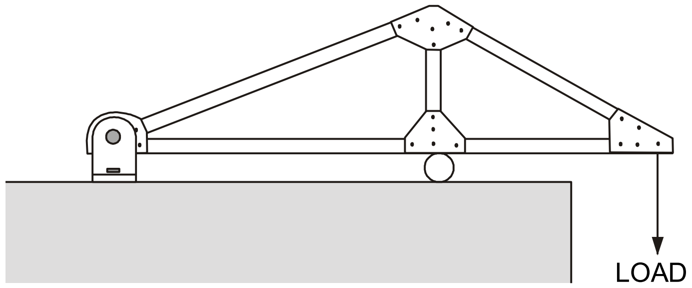


b)



MOMENT OF A FORCE

The moment of a force is the turning effect of that force when it acts on a body

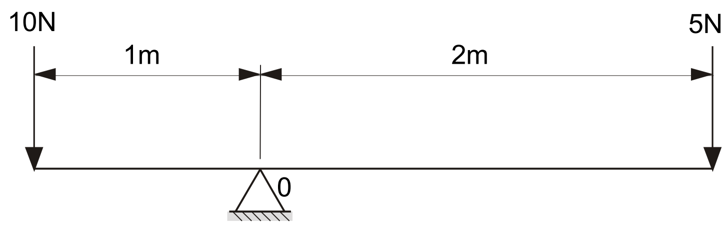


The load acting on the frame structure above will have a turning effect on the structure.

The Principle of Moments states that if a body is in Equilibrium the sum of the clockwise moments is equal to the sum of anti-clockwise moments.

Worked Examples

Example 1



$$\sum CwM = \sum ACwM$$

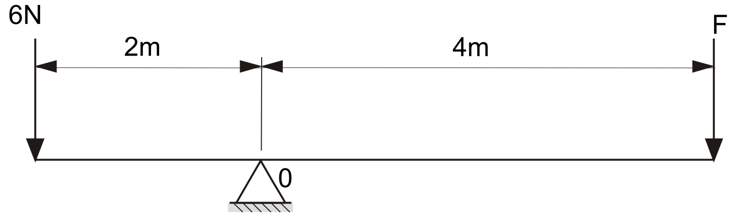
$$F \times d = F \times d$$

$$5 \times 2 = 10 \times 1$$

THE ABOVE EXAMPLE IS IN EQULIBRUIM

We can use this principle to find an unknown force or unknown distance.

Example 2



CwM = ACwM

(F x 4) = (6 x 2)

4F = 12

F = 12/4

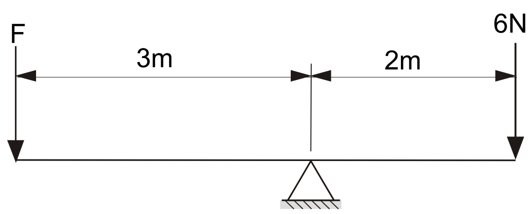
F = 3N

Assignments: Moments

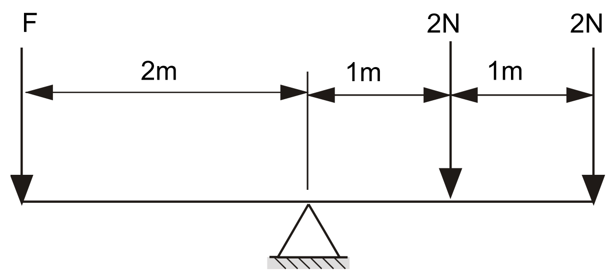
The beams shown below are in equilibrium. Find the unknown quantity for each arrangement.

Question 1

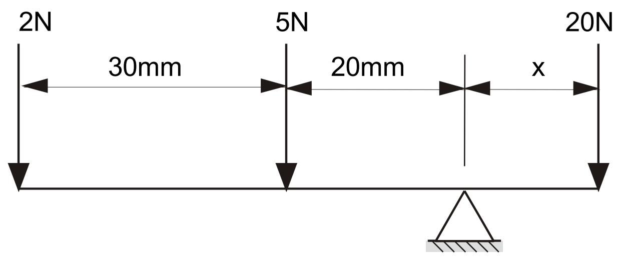
a)



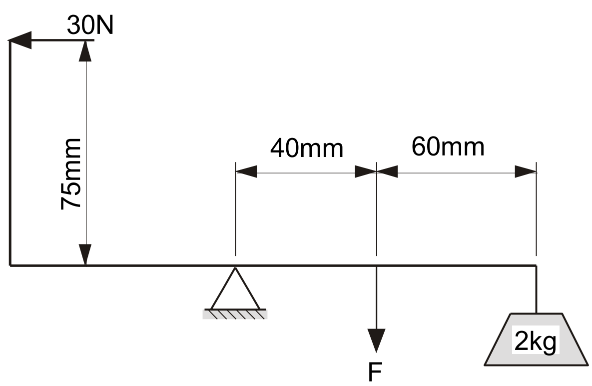
b)



c)



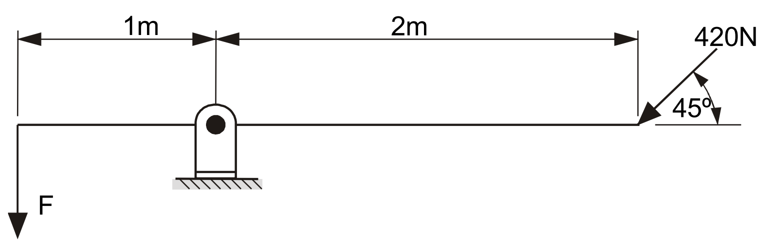
d)



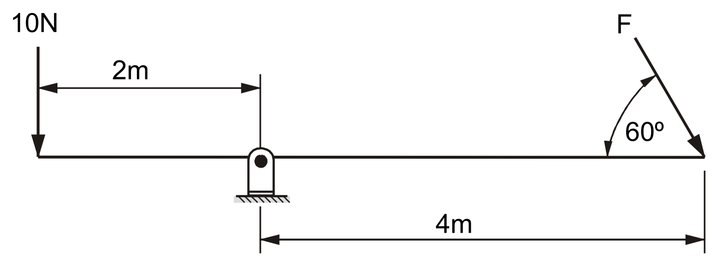
The following beams, in equilibrium, have inclined forces. Find the unknown quantity.

Question 2

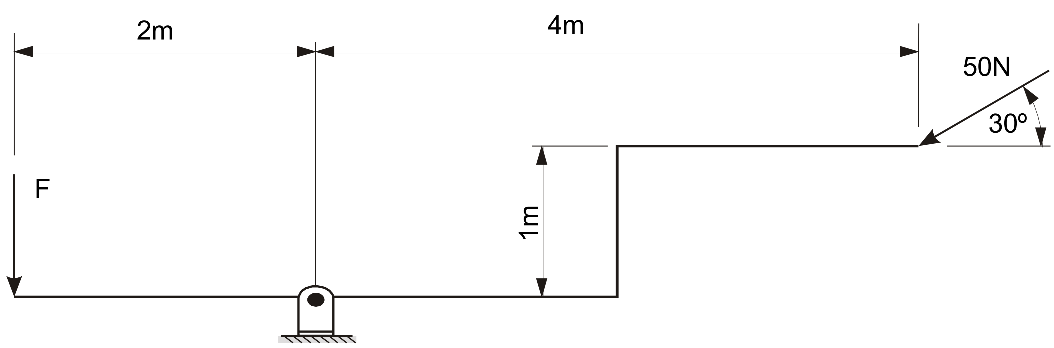
a)



b)



c)



BEAM REACTIONS

We are now going to study beams with external forces acting on them. We shall resolve forces into their components and use moments to find the support reactions.

Definitions of some of the terms you have met already:

RESULTANT	The resultant is that single force that replaces a system of forces.
EQUILIBRIUM	Equilibrium is the word used to mean balanced forces.

Conditions of Equilibrium

1. Clockwise Moments = Anti-Clockwise Moments

$$\sum CwM = \sum ACwM$$

2. The sum of the forces in the vertical direction equals zero.

$$\sum F_{vertical} = 0$$

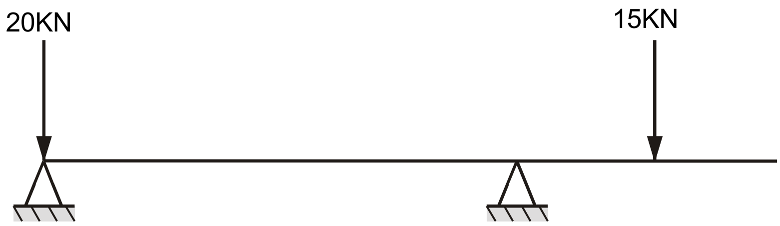
3. The sum of the forces in the horizontal direction equals zero.

$$\Sigma F_{horizontal} = 0$$

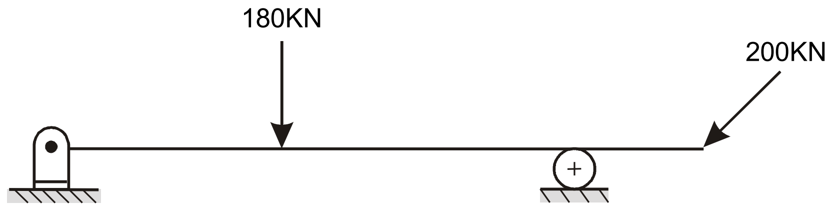
Beam Reactions

A beam is usually supported at two points. There are two main ways of supporting a beam -

1. Simple supports (knife edge)



2. Hinge and roller

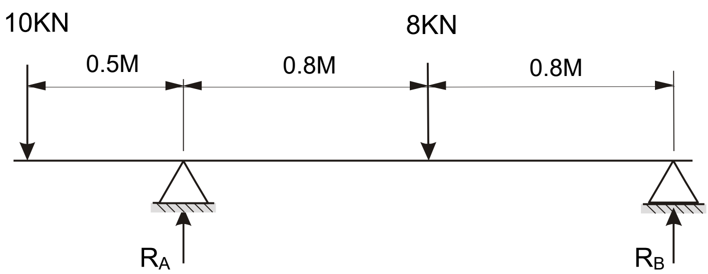


Worked Examples

1. SIMPLE SUPPORTS

Simple supports are used when there is no sideways tendency to move the beam.

Consider this loaded beam, "simply" supported.



1. The forces at the supports called reactions, always act vertically.

2. The beam is in equilibrium; therefore the conditions of equilibrium apply.

The value of Reactions **R_A** and **R_B** are found as follows.

Take moments about R_A

$$CwM = ACwM$$

$$(8 \times 0.8) = (R_b \times 1.6) + (10 \times 0.5)$$

$$6.4 = 1.6R_b + 5$$

$$1.6R_b = 6.4 - 5$$

$$R_b = 1.4/\ 1.6$$

$$R_b = 0.88kN$$

To find R_A

$$\sum Fv = 0$$

$$\text{Upwards forces} = \text{Downwards forces}$$

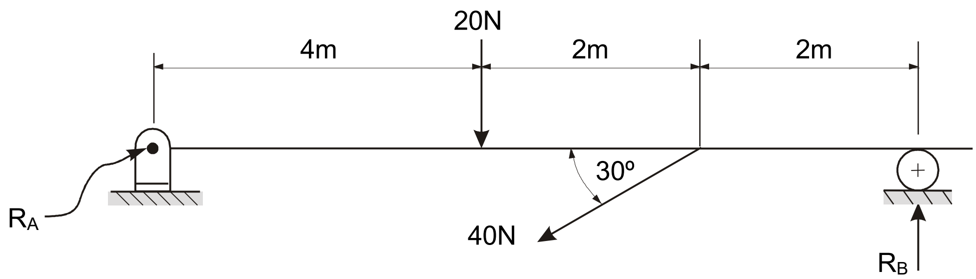
$$R_a + R_b = 10 + 8$$

$$R_a = 18 - R_b$$

$$R_a = 18 - 0.88$$

2. HINGE AND ROLLER SUPPORTS

Hinge and roller supports are used when there is a possibility that the beam may move sideways.



Note: The reaction at a roller support is always at right angles to the surface. The direction of $\mathbf{R_A}$ is assumed. If any of the components work out as negative values then the direction will be opposite the assumed direction.

The reaction at the hinge support can be any direction.

(Find the two components of the hinge reaction, then the resultant)

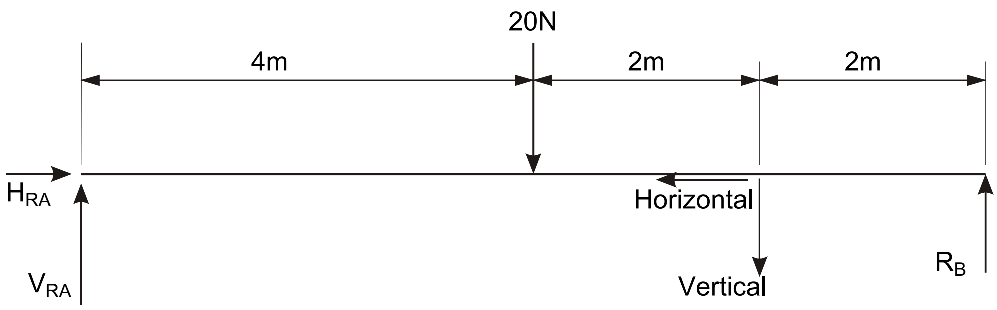
There are three unknown quantities above: -

1. The magnitude of Reaction $\mathbf{R_B}$.

2. The magnitude of Reaction $\mathbf{R_A}$.

3. The direction of Reaction $\mathbf{R_A}$.

Redraw as a free-body diagram showing vertical and horizontal components of the forces.



The vertical and horizontal components of the 40N force are found.

$$V = \sin 30^0 \times 40 \qquad H = \cos 30^0 \times 40$$

$$V = 20N \qquad H = 34.64N$$

To find $\mathbf{R_B}$ -take moments about $\mathbf{V_{RA}}$, this eliminates one of the unknown vertical forces.

$$CWM = ACWM$$
$$20\times4 \;+\; 20\times6 \;=\; R_B\times8$$
$$8R_B = 200$$
$$R_B = \frac{200}{8}$$
$$R_B = 25N$$

Vertical forces

$$\Sigma F_Y = 0$$
$$V_{RA} + R_B = 20 + 20$$
$$V_{RA} = 40 - R_B$$
$$V_{RA} = 40 - 25$$

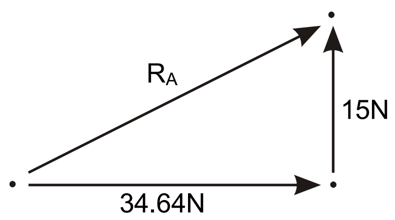
$$V_{RA} = 15N$$

Horizontal forces

$$\Sigma Fx = 0$$
$$H_{RA} - 34.64 = 0$$

$$H_{RA} = 34.64N$$

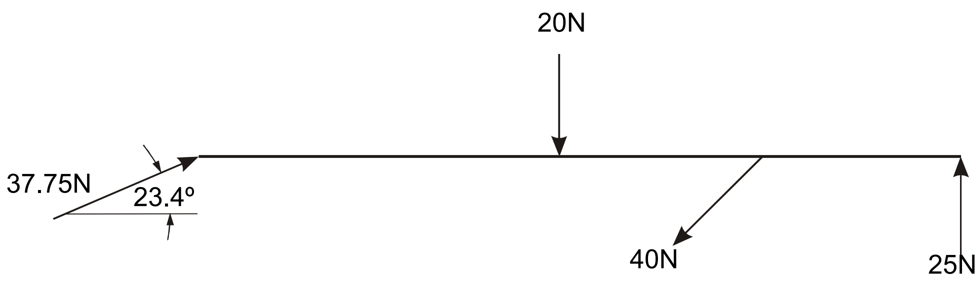
Use V_{RA} and H_{RA} to find R_A



$$R_A = \sqrt{15^2 + 34.64^2}$$
$$= \sqrt{225 + 1199.93}$$
$$= 37.75N$$

Find direction of R_A

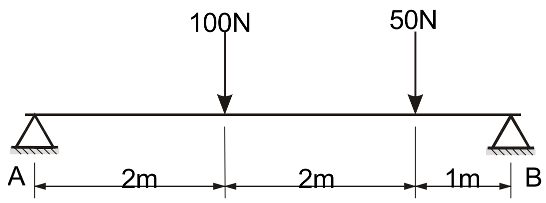
$$\text{t n}\,a = \frac{VR_A}{HR_A}$$
$$= \frac{15}{34.64}$$
$$= 23.4^\circ$$



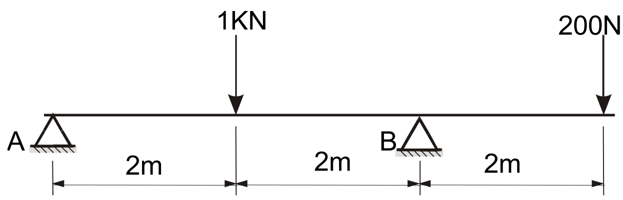
Assignments: Beam Reaction

1 Find the reactions at supports A and B for each of the loaded beams shown below.

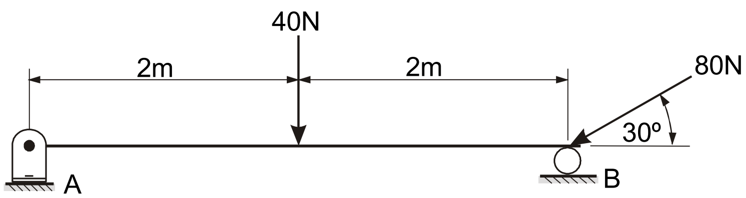
a)



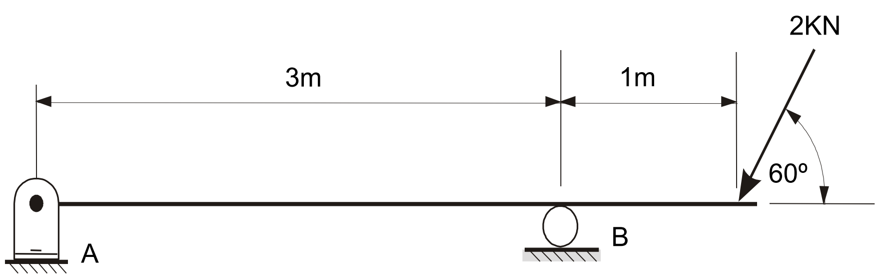
b)



c)



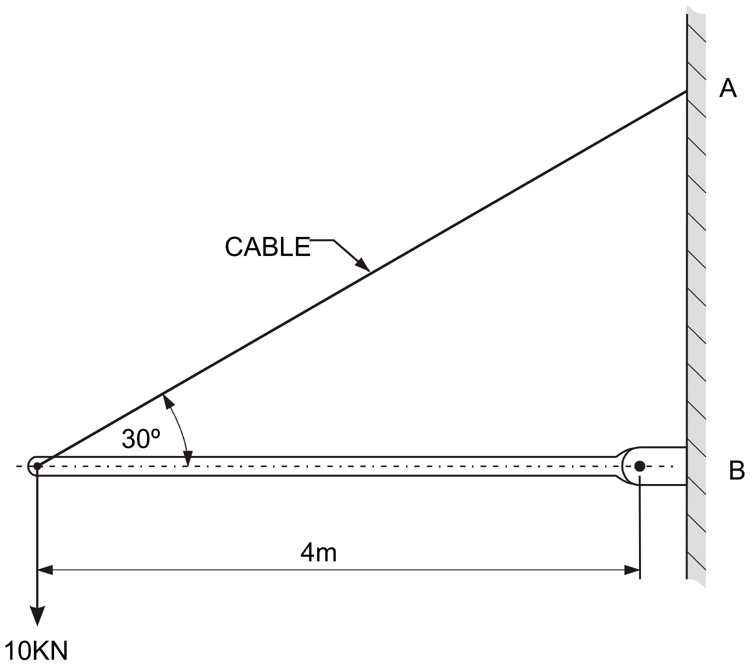
d)



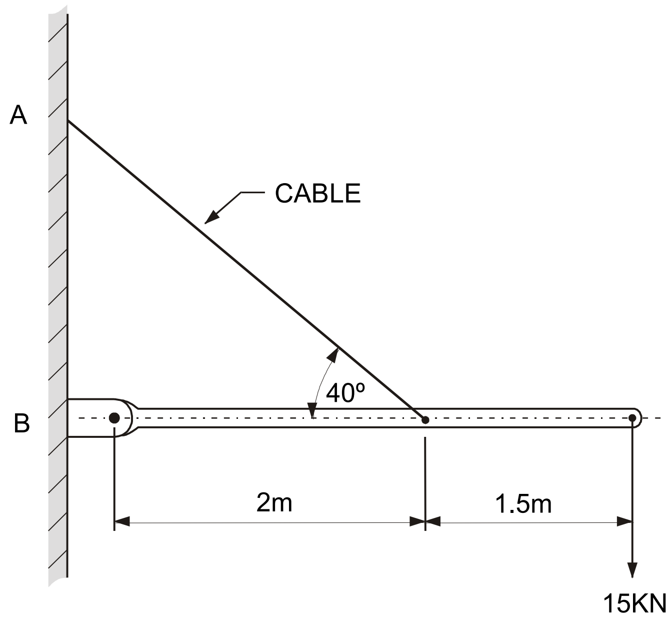
Task - Beam Reaction (continued)

2) Find the reactions at supports A and B for each of the loaded beams shown below.

a)

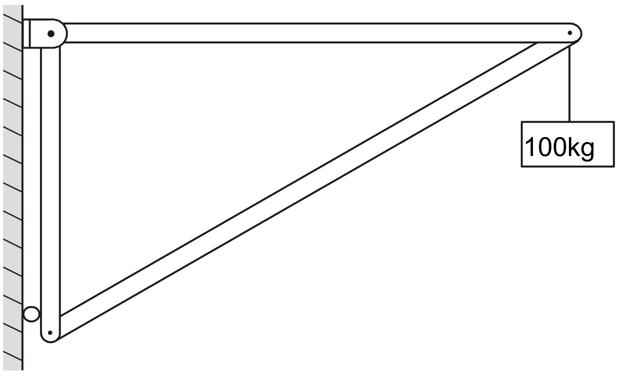


b)



FRAMED STRUCTURES

A frame structure is an assembly of members and joints (usually called Nodes) which is designed to support a load. Examples of frame structures include roof trusses, bridges, pylon towers.



The members in this framed structure can be as ties or struts, depending on the type of force they support.

STRUTS AND TIES

When solving problems in frame structures you will be required to determine the Magnitude and Nature of the forces in the members of the frame. That is, determine, in addition to the size of the force in the member, whether the member is a Strut or a Tie.

Strut

Members that are in compression, due to external forces trying to compress them, are known as Struts.

Tie

Members that are in tension, due to external forces trying to pull them apart, are known as Ties.

Nodal Analysis

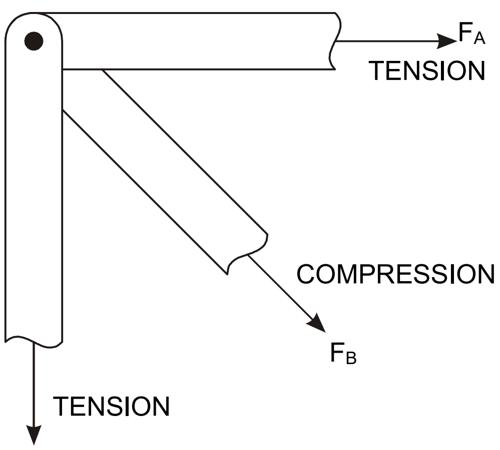
There are several methods of solving frame structure problems. The method we shall use is called Nodal Analysis.

Any joint where members meet is known as a node.

This method relies on the fact that if structures are in equilibrium then each node will be in equilibrium. The sum of the forces acting on any the node will equal zero.

Nodes

The members are either in compression (strut) or tension (tie). They can be represented at the node as shown below –



F_B is under compression and pushes into the node, **F_A** is under tension and pulls away from the node.

Conditions of Static Equilibrium

SUM OF THE MOMENTS = 0	$\Sigma M_O = 0$
SUM OF THE VERTICAL FORCES = 0	$\Sigma F_V = 0$
SUM OF THE HORIZONTAL FORCES = 0	$\Sigma F_H = 0$

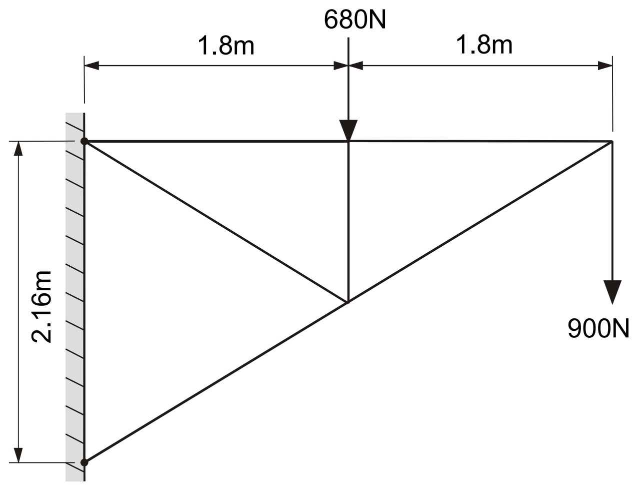
When solving frame structures some analysis of the structure is required to determine the best starting point and which of the conditions of static equilibrium to apply first.

Solving Simple Frame Structures

To help solve frame structure problems there are some simple rules to follow depending on the type of structure.

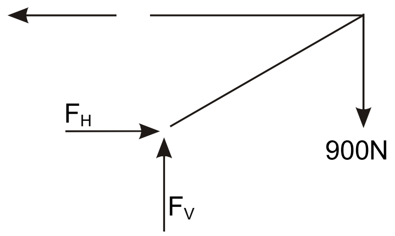
Cantilevered frame structures
For this frame structure it is not necessary to use moments to help find the forces in the members. .

The node with the 900N load acting has only one unknown vertical component.



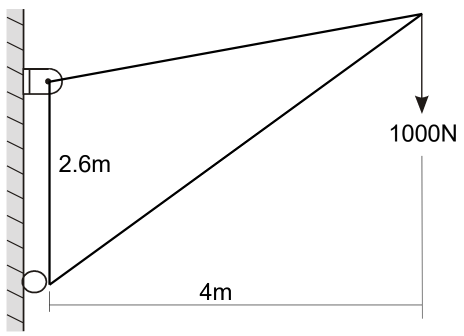
As all vertical forces acting on the node must equal zero then **F_V** must equal 900N.

By using trigonometry it is now possible to find the other forces acting on this node.

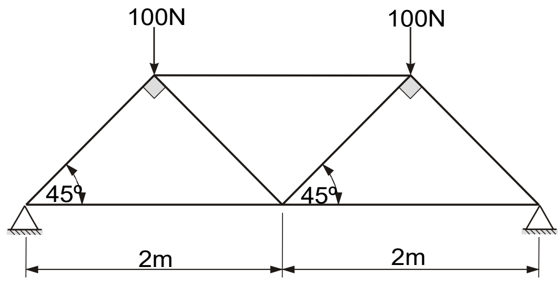


In this example all nodes have more than one unknown force or component of a force.

To solve this frame structure take moments about the top support to find the reaction force at the roller support.



Truss frame structures

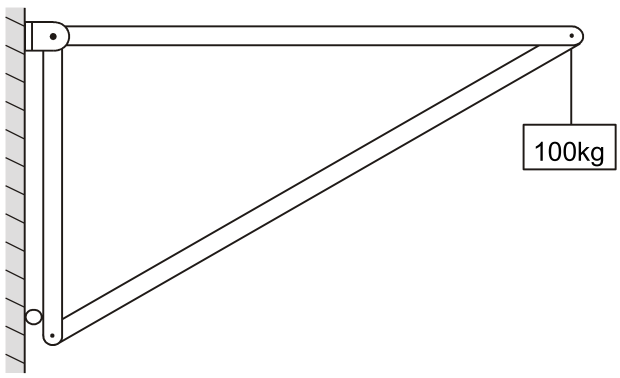


Treat structure like this as a beam and use moments to find reactions. Until the reaction forces are found all nodes have more than one unknown force or component.

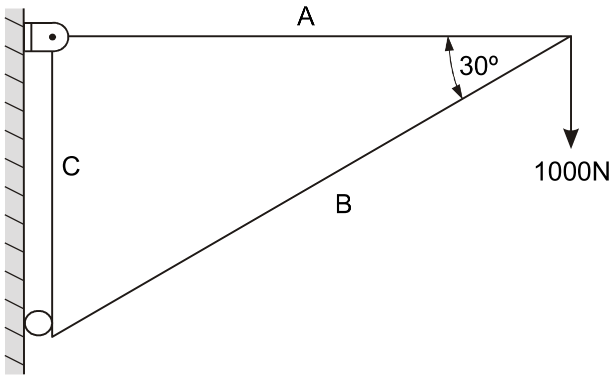
Then start analysis at $\mathbf{R_1}$ or $\mathbf{R_2}$, which will now have only one unknown component.

Worked Example

For the crane shown below we shall find the forces in the members and the reaction forces at the wall.



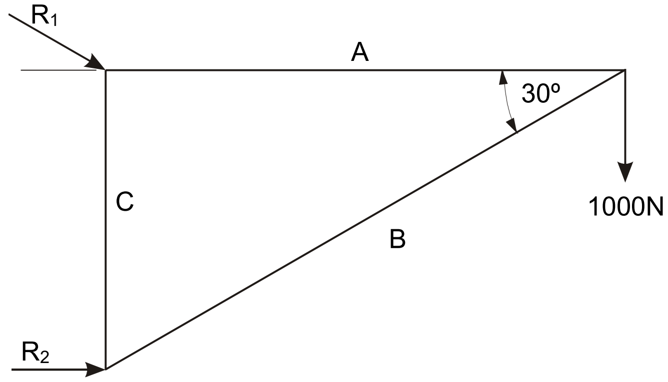
Draw a free body diagram of the frame structure.



The framework is supported at two points. The hinge support at the top is being pulled away from the wall. $\mathbf{R_1}$ will act against this pull and keep the hinge attached to the wall. The roller support is being pushed into the wall. $\mathbf{R_2}$ will act against this force and in the opposite direction as shown below.

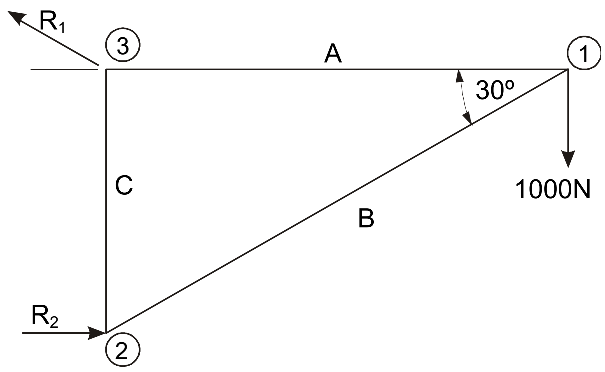
As member **B** is acting on a roller then $\mathbf{R_2}$ will be at 90° to member **C**

As $\mathbf{R_1}$ is acting at a hinge support there will be a vertical and horizontal component. At this stage guess the direction of $\mathbf{R_1}$.



If a wrong assumption about the member being in tension or compression or the direction of a force is made then a negative value will be produced from the calculation. If this is the case the direction of the force is simply reversed

To find the forces in the members and the reactions at the supports, study each of the structure nodes separately.



Each node is in equilibrium so we can use the conditions of equilibrium.

SUM OF THE VERTICAL FORCES = 0 $\Sigma F_V = 0$

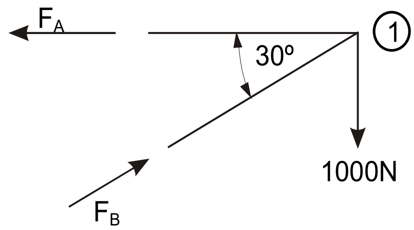
SUM OF THE HORIZONTAL FORCES = 0 $\Sigma F_H = 0$

SUM OF THE MOMENTS = 0 $\Sigma M_O = 0$

In this sort of problem where the structure is cantilevered out from the wall, it is normally solvable without considering the moments. Start by finding a node that has only one unknown vertical or horizontal component.

Node 1

The forces acting on node 1 are shown below.

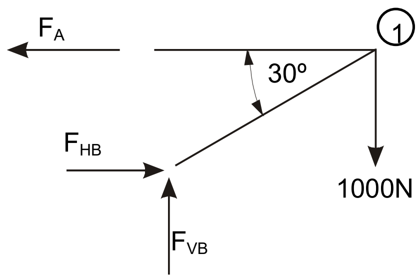


Note - an assumption has been made about the direction of $\mathbf{F_A}$ and $\mathbf{F_B}$.

$\mathbf{F_A}$ is assumed to be in tension and acting away from the node 1.

$\mathbf{F_B}$ is assumed to be in compression and acting in towards the node 1.

Split $\mathbf{F_B}$ into its horizontal ($\mathbf{F_{HB}}$) and vertical ($\mathbf{F_{VB}}$) components.

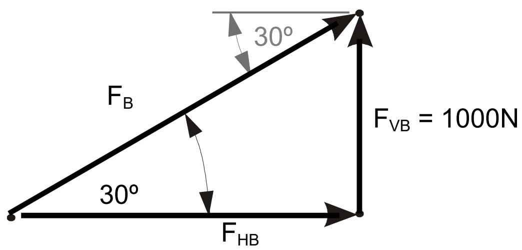


Apply a condition of static equilibrium -

$$\Sigma F_V = 0$$

As the sum of Vertical forces is equal to zero then the vertical component of **F_B** (**F_{VB}**) is 1000N acting up.

From this we can find **F_B** - we can redraw **F_B** and the two components **F_{HB}** and **F_{VB}** to form a triangle as shown below.



$$u \sin g - \sin \theta = \frac{opp}{hyp}$$

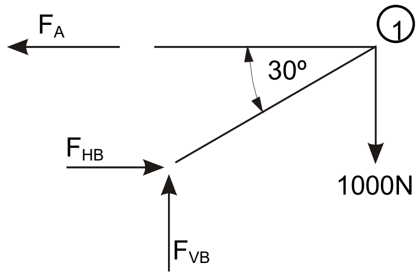
$$\sin 30° = \frac{F_{VB}}{F_B}$$

$$F_B = \frac{F_{VB}}{Sin 30°}$$

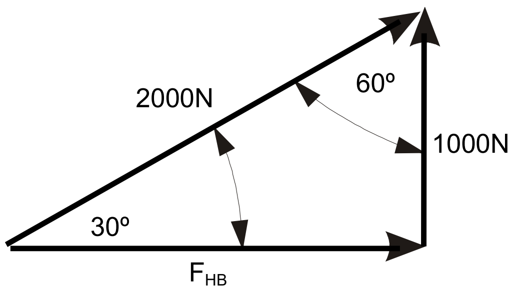
$$= \frac{1000}{0.5}$$

$$= 2000N$$

The force in member **A** (**F_A**) will equal and opposite to **F_{HB}**.



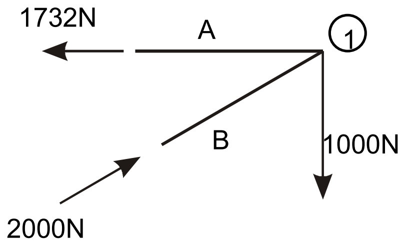
Find $\mathbf{F_{HB}}$ -



$$
\begin{aligned}
\cos 30° &= \frac{F_{HB}}{F_B} \\
F_{HB} &= \cos 30° \times F_B \\
&= 0.87 \times 2000 \\
&= 1732\,N
\end{aligned}
$$

From above $\mathbf{F_A}$ = 1732N

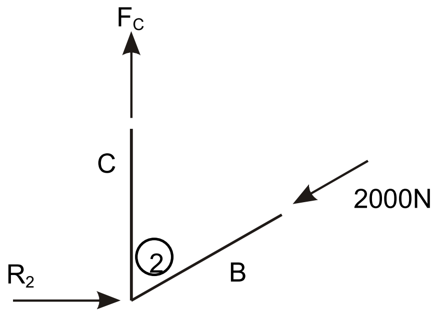
This diagram shows all the forces acting at node 1.



Worked Example (continued)

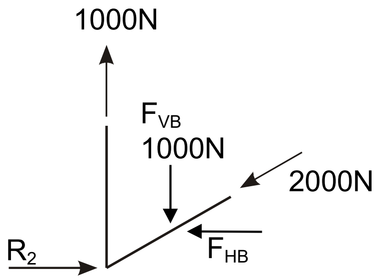
Node 2

There are three forces acting at node 2.



During analysis of node 1, $\mathbf{F_B}$ was found to be 2000N and was not negative so the assumed direction towards the node (compression) was correct.

In node 1, $\mathbf{F_B}$ is shown to be acting as a compressive force towards the node with a magnitude of 2000N.



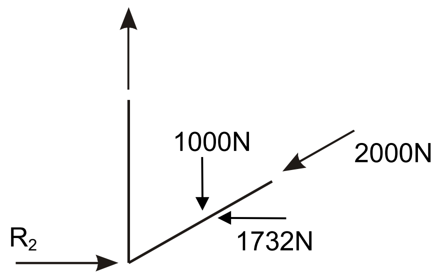
$\mathbf{F_C}$ will be equal and opposite to the Vertical component of $\mathbf{F_B}$ ($\mathbf{F_{VB}}$).

From analysis of node 1 $\mathbf{F_{VB}}$ was equal to 1000 N therefore -

$$\mathbf{F_C} = 1000 \text{ N}$$

As $\mathbf{R_2}$ is a roller support the reaction will be at 90° to the surface.

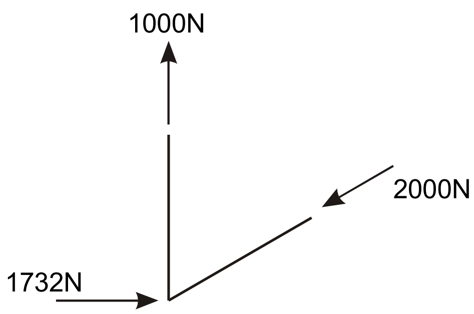
$\mathbf{R_2}$ will be equal to the Horizontal component of $\mathbf{F_B}$.



From above $\mathbf{H_B}$ was equal to 1732 N therefore -

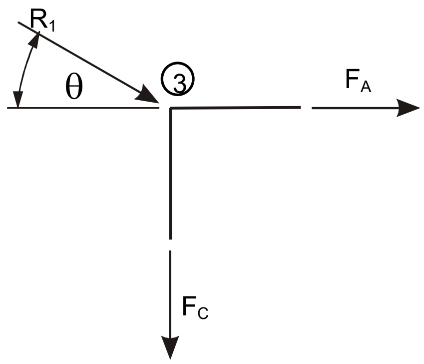
$$\mathbf{R_2} = 1732\text{N}$$

The forces acting at node 2 are shown below.



Node 3

Node 3 has three forces acting on it.

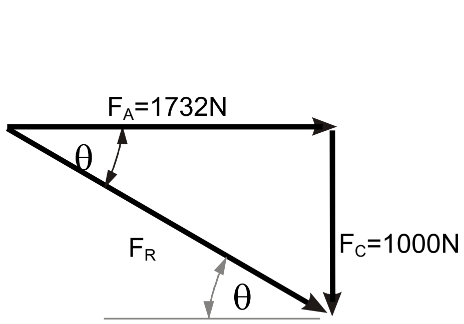


$\mathbf{R_1}$ will be equal and opposite to the resultant of $\mathbf{F_A}$ and $\mathbf{F_C}$.

From analysis of node 1 $\mathbf{F_A}$ was found to be 1732N.

From analysis of node 2 $\mathbf{F_C}$ was found to be 1000N

Find resultant of $\mathbf{F_A}$ & $\mathbf{F_C}$ - $\mathbf{F_R}$

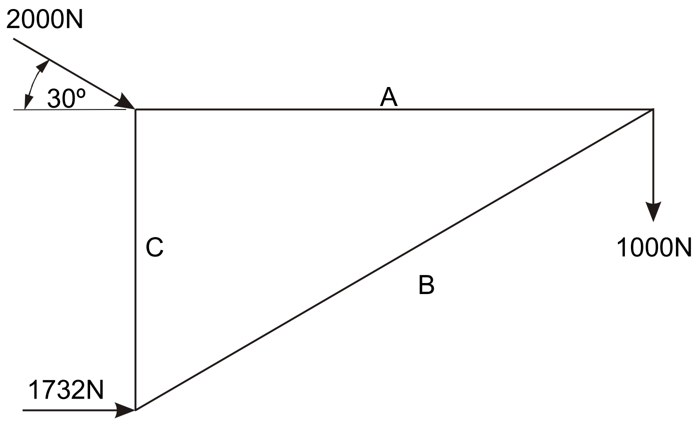


$$F_R = \sqrt{F_A^2 + F_C^2}$$
$$= \sqrt{1732^2 + 1000^2}$$
$$= 2000N$$

Find angle of resultant (θ) -

$$t\ n\theta = \frac{F_C}{F_A}$$
$$= \frac{1000}{1732}$$
$$= 0.58$$
$$\theta = 30^\circ$$

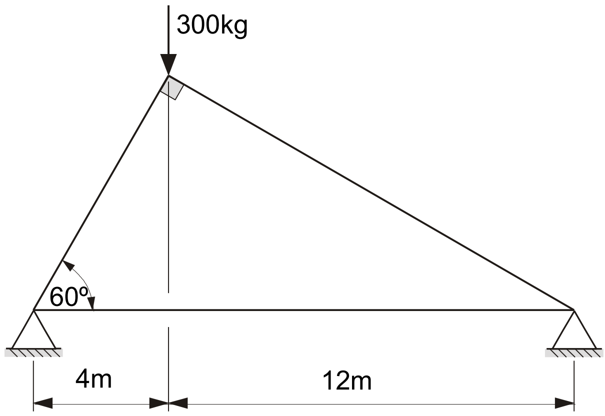
The task was to find the forces in the members and the reactions forces at the supports. The results can be shown as below -



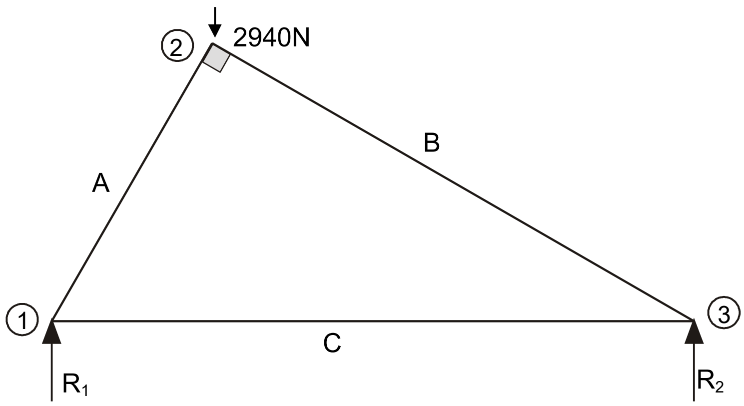
MEMBER	FORCE	TYPE OF FORCE
A	1732N	TENSION
B	2000N	COMPRESSION
C	1000N	TENSION

Assignment 1

Find the reactions R_1 and R_2 and the forces for the members in the frame structure shown below.



The free body diagram with the nodes and members labelled is given below.



In this type of problem where the structure is acting as a beam, find R1 and R_2 using moments.

Take moments about R_1 -

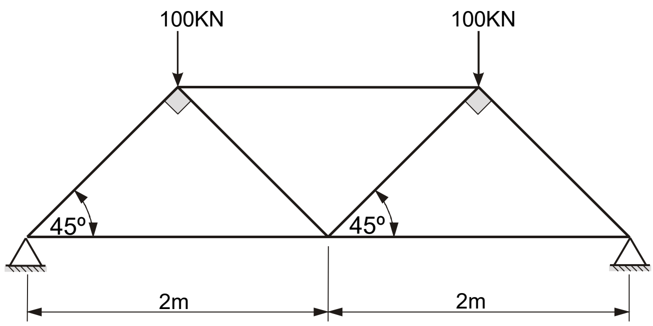
$$\Sigma M_{R1} = 0$$
$$(2940 \times 4) + (-R_2 \times 16) = 0$$
$$R_2 \times 16 = 11760$$
$$R_2 = \frac{11760}{16}$$
$$R_2 = 735N$$

Now complete this task and find R_1 and the forces in A, B and C.

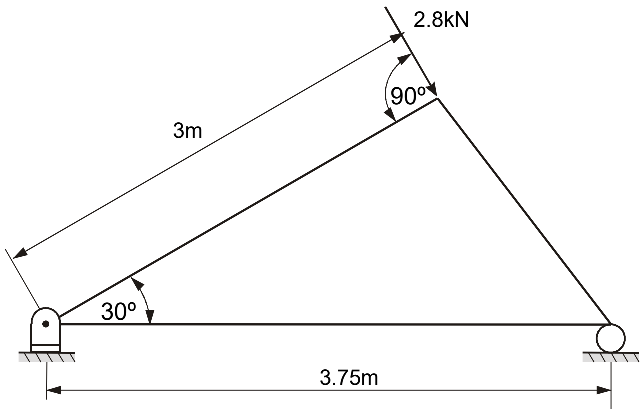
Assignment 2

Find the reactions, the magnitude and nature of the forces in the members of the frame structures shown below.

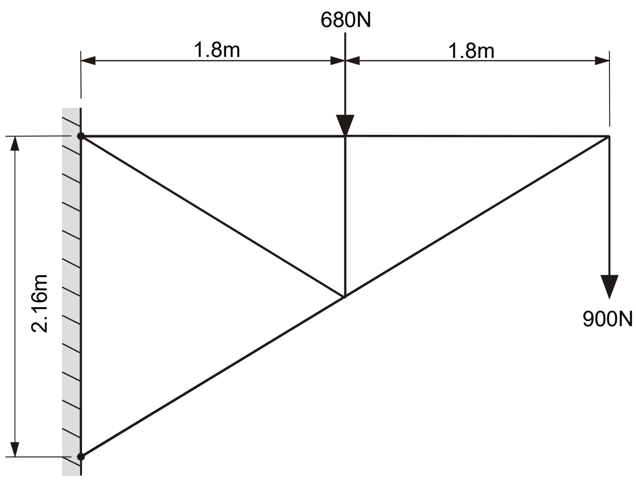
1)



2)

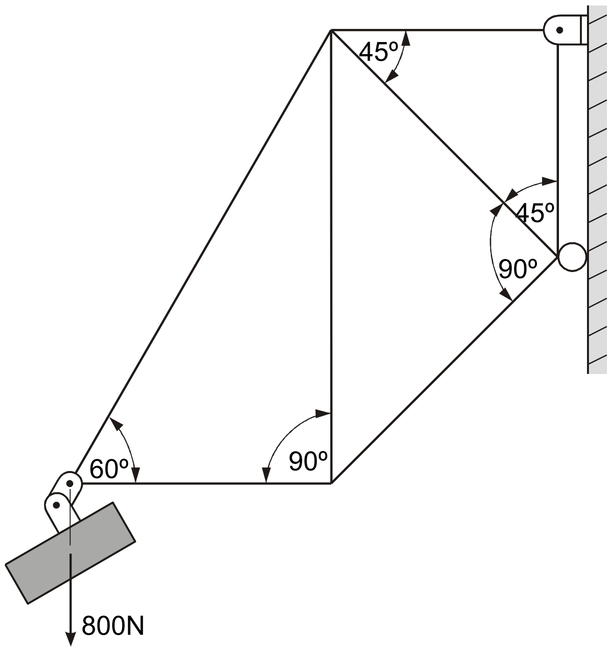


3)



Assignment 3

Calculate reactions and find the magnitude and nature of forces in the members in the lighting gantry structure shown below.



PROPERTIES OF MATERIALS

It is not only the shape of a structure that will influence its overall performance but also the material that each member in the structure is made from. If any one member was to fail within the structure itself, it would create a domino effect on the other members and ultimately the structure would collapse.

In order to select a material for a particular purpose the structural engineer must consider a range of materials, all with different properties. He/she will choose the material that is best suited to the job in hand.

The most common properties to be considered include:

1. **STRENGTH** - the ability of a material to resist force. All materials have some degree of strength - the greater the force the material can resist, the stronger the material. Some materials can be strong in tension but weak in compression, for example mild steel. The converse can also be true, as is the case with concrete, which is strong in compression but weak in tension. Hence, the reason that concrete is often reinforced with mild steel.

2. **ELASTICITY** - the ability of a material to return to its original shape or length once an applied load or force has been removed. A material such as rubber is described as elastic because it can be stretched but when it is released it will return to its original condition.

3. **PLASTICITY** - the ability of a material to change its shape or length under a load and stay deformed even when the load is removed.

4. **DUCTILITY** - the ability of a material to be stretched without fracturing and be formed into shapes such as very thin sheets or very thin wire. Copper, for example, is very ductile and behaves in a plastic manner when stretched.

5. **BRITTLENESS** - the property of being easily cracked, snapped or broken. It is the opposite of ductility and therefore the material has little plasticity and will fail under loading without stretching or changing shape. Cast iron and glass are obvious examples of materials that are brittle.

6. **MALLEABILITY** - the ability of a material to be shaped, worked or formed without fracturing. It is closely related to the property of plasticity.

7. **TOUGHNESS** - the ability to absorb a sudden sharp load without causing permanent deformation or failure. Tough materials require high elasticity.

8. **HARDNESS** - the ability to resist erosion or surface wear. Hard materials are used in situations where two surfaces are moving across or over each other.

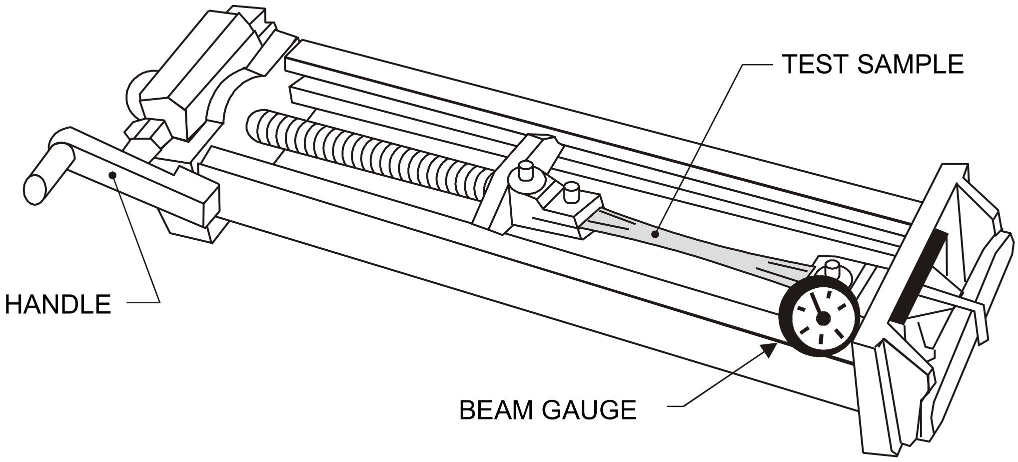
MATERIALS TESTING

In order to discover the various properties of a material we must carry out material tests. There are many different types of tests available but the most common is the tensile test. As the name suggests the material is subjected to a tensile force or in other words, the material is stretched or pulled apart.

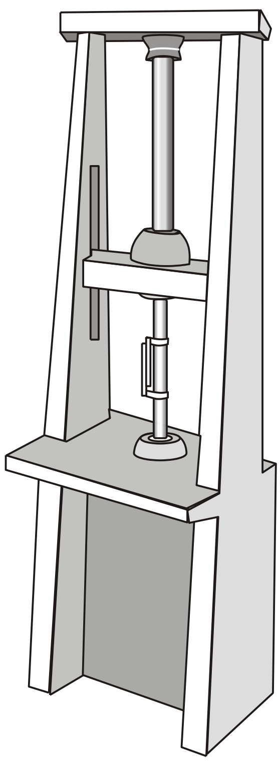
Results from tensile tests allow us to determine the following properties:

1. The elasticity of a material
2. The plasticity or ductility of the material
3. The ultimate tensile strength of the material.

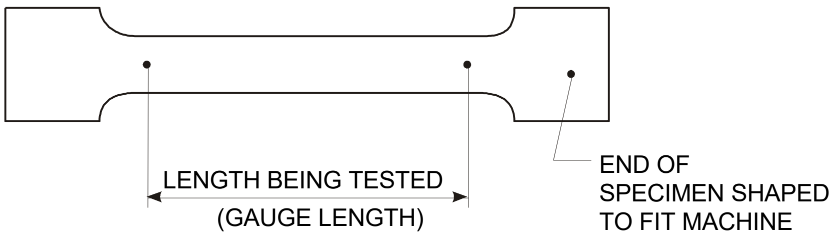
A tensometer or tensile testing machine is designed to apply a controlled tensile force to a sample of the material. A tensometer that might be found in schools is shown below.



More sophisticated tensometers are available and are commonly used in industry. The main advantage of these machines is that they are able to plot a graph of how the material behaves during the test. A Hounsfield tensometer is shown below.

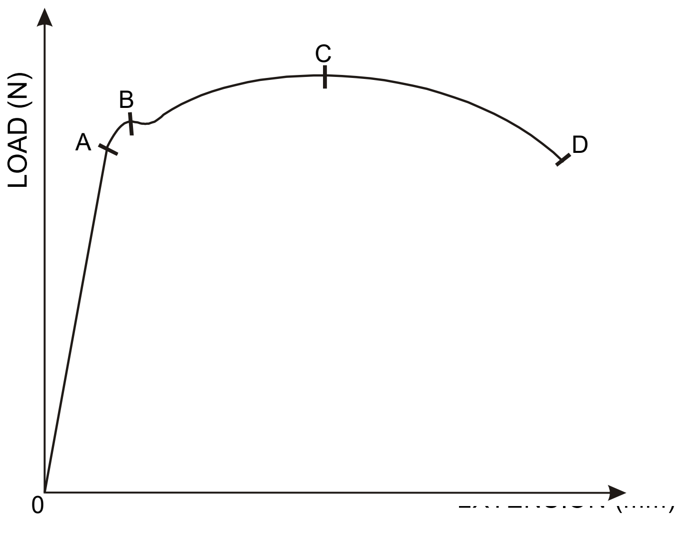


In order for tests to be carried out on a consistent basis, the shape of the specimen to be tested must conform to British Standards. The test sample is prepared to have a thin central section of uniform cross-section. A typical test specimen is shown below.



The principle of tensile testing is very simple. As the force is applied to the specimen, the material begins to stretch or extend. The tensometer applies the force at a constant rate and readings of force and extension are noted until the specimen finally breaks. These readings can be plotted on a graph to show the overall performance of the material.

The results of a typical tensile test for a sample of mild steel are shown.



The shape of the graph is very important and helps us predict how the material will behave or react under different loading conditions.

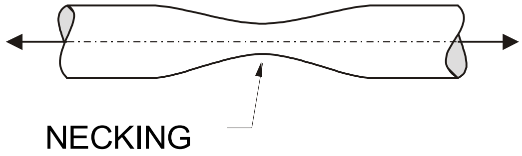
Between points 0 and 'A' the material behaves elastically and this part of the graph is known as the elastic region. This means that the material stretches under the load but returns to its original length when the load is removed. In fact, the force and extension produced are proportional and this part of the graph will be a straight line. This relationship is known as Hooke's Law and is very important to structural engineers.

'A' is called the Limit of Elasticity and any loading beyond this point results in plastic deformation of the sample.

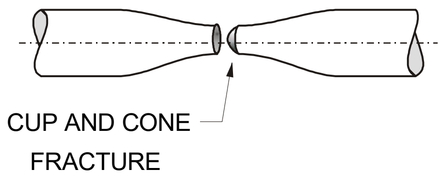
'B' is called the yield point and a permanent change in length results even when the load is removed. Loading beyond this point results in rapidly increasing extension.
Between points 'B' and 'D' the material behaves in a plastic or ductile manner.

At point 'C' the maximum or ultimate tensile force that the material can withstand is reached.

Between 'C' and 'D' the cross-sectional area of the sample reduces or 'necks'.



‘Necking’ reduces the cross-sectional area of the specimen, which in turn reduces the strength of the sample. The sample eventually breaks or fractures at point ‘D’. The shape of a typical fractured specimen is shown below.



Assignments: Tensile Testing

You intend to carry out a tensile test on a piece of soft copper to establish some of its physical properties and characteristics.

a) Explain, briefly, how you would carry out such a test and what equipment would be required to do this.
b) Sketch a typical specimen test piece that would be used in such a test. Indicate clearly two important physical readings that would have to be taken at the beginning of the test.
c) State one property that may be established from the results of the test.

2.

a) Sketch a typical load extension graph for a mild steel specimen that has undergone a tensile test to destruction. On the graph, indicate the following points:

Yield point

Elastic region

Plastic region

Breaking point

Maximum load

b) State three properties that can be compared between two specimens of identical shape and size, one made from mild steel and the other from annealed copper, if they are both tested to destruction using tensile tests.

3.

a) Explain the difference between plastic deformation and elastic deformation with respect to engineering materials.

b) Give an example of two pieces of modern engineering design: one designed to display elastic deformation; the other to display plastic deformation.

4. A tensile test on an unknown material produced the following results.

Force kN	4.45	8.9	17.8	26.7	35.6	44.5	53.4	62.3
Extension mm	0	1.2	2.3	4.5	4.6	5.7	7.7	11

Plot a graph of the load against the extension.

A mild steel specimen 25 mm in diameter and 250 mm long was subjected to a gradually increasing tensile load until finally the specimen snapped. The following results were obtained.

Force kN	20	40	60	80	100	120
Extension mm	0.048	0.097	0.142	0.196	0.241	0.287
Force kN	140	160	170	180	190	
Extension mm	0.343	0.39	0.42	0.46	0.52	

a) Plot a graph of load against extension.

b) On the graph indicate three important points.

The results from a tensile test to destruction are shown below.

Force kN	50	100	150	200	250	260	270
Extension mm	0.0007	0.0014	0.0022	0.0029	0.0036	0.004	0.005
Force kN	280	290	300	290	280	270	260
Extension mm	0.0066	0.0094	0.02	0.0238	0.0256	0.028	0.03

a) Plot a graph of load against extension.

b) On the graph indicate three important points.

c) With reference to the graph, describe the effect of gradually increased tensile load on the specimen.

The results of performing a tensile test on a specimen are shown below.

Force kN	Extension mm	Force kN	Extension mm
10	0.02	70	0.552
20	0.036	71	0.678
30	0.056	72	0.822
40	0.076	73	1.02
50	0.095	72	1.22
55	0.104	71	1.35
60	0.12	70	1.48
64	0.202	69	1.59
67	0.299	68	1.66
68	0.391	65	1.7

a) Plot a graph of load against extension.

b) With reference to the graph, describe the properties of the material tested.

Results from a tensile test to destruction are shown below.

Force kN	Extension mm	Force kN	Extension mm
16.5	0.04	81.3	0.3
25.5	0.062	81.2	0.3
33.2	0.0805	80	0.4
38.2	0.0927	80.8	0.5
44	0.107	82	0.75
55	0.134	75.5	20.8
77.7	0.188	72	2.38
80	0.24	67.6	2.75
80.7	0.25	61	3.17
81.5	0.252	54	3.5

a) Plot a graph of load against extension.

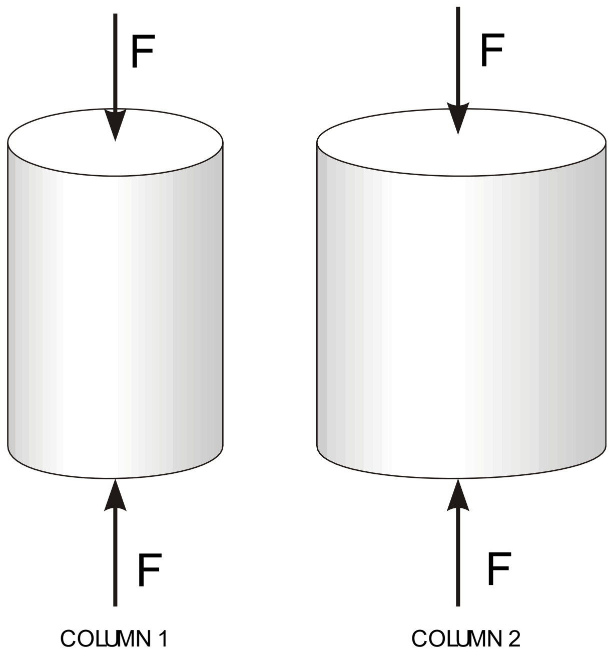
c) With reference to the graph, describe the properties of the material tested.

STRESS STRAIN GRAPHS

Far more useful to an engineer than a load extension graph is a stress strain graph. This in many ways resembles a load extension graph but the data in this form can be interpreted more easily in design situations. First let us examine what is meant by stress and strain.

Stress

When a direct force or load is applied to the member of a structure, the effect will depend on the cross-sectional area of the member. Lets look at column 1 and 2 below. Column 2 has a greater cross-sectional area than column 1. If we apply the same load to each column, then column 1 will be more effected by the force.



The effect that the force has on a structural member or element is called STRESS. This is calculated using the formula:

$$Stress = \frac{Force}{Area}$$

$$\sigma = \frac{F}{A}$$

where Force is measured in Newtons (N) and Area is the cross-sectional area measured in mm2. Stress therefore is measured in N/mm2 and is denoted by the greek letter sigma (σ).

Worked examples: Stress

A square bar of 20 mm x 20 mm cross-section is subjected to a tensile load of 500 N. Calculate the stress in the bar.

$$Stress = \frac{Force}{Area}$$

$$\sigma = \frac{F}{A}$$

$$\sigma = \frac{500}{400}$$

$$\sigma = 1.25N \quad mm^2$$

Stress in the bar = 1.25 N/mm^2

A column of section 0.25 m2 is required to act as a roof support. The maximum allowable working stress in the column is 50 N/mm2. Calculate the maximum compressive load acting on the column.

$$Stress = \frac{Force}{Area}$$

$$Force = Stress \times Area$$

$$Force = 50 \times 0.25 \times 10^6$$

$$Force = 12.5MN$$

Maximum compressive load acting on the column = 12.5 MN

The stress in a steel wire supporting a load of 8 kN should not exceed 200 N/mm2. Calculate the minimum diameter of wire required to support the load.

$$Stress = \frac{Force}{Area}$$

$$Area = \frac{Force}{Stress}$$

$$Area = \frac{8000}{200}$$

$$Area = 40mm^2$$

$$Area = \frac{\pi d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}}$$

$$d = \sqrt{\frac{4 \times 40}{\pi}}$$

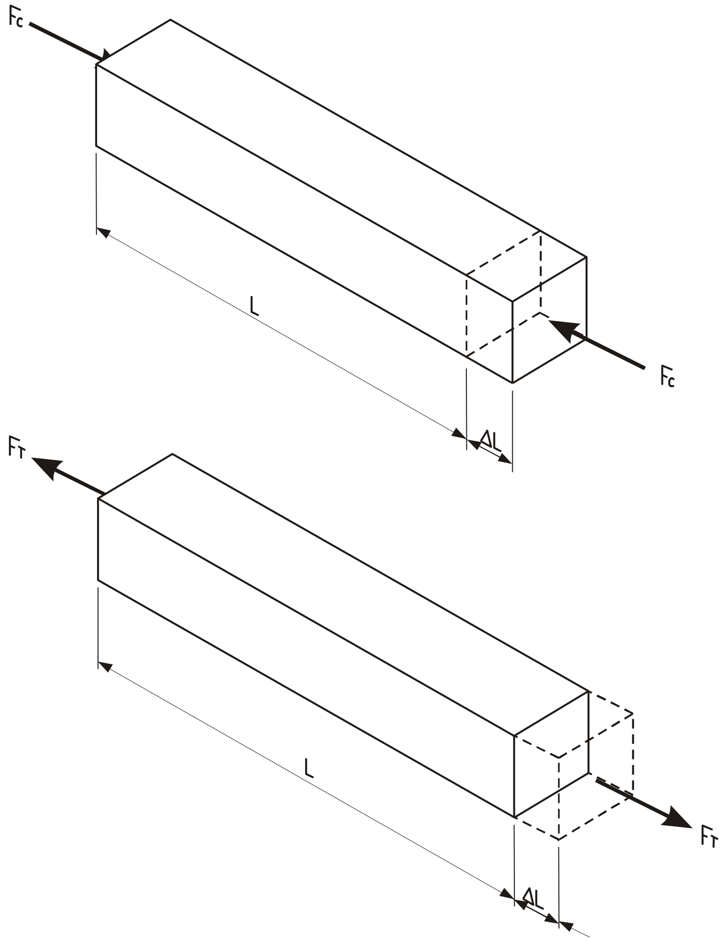
$$d = 7.14mm$$

Minimum diameter of wire required to support load = 7.14 mm

Strain

The result of applying a load or force to a structural member is a change in length. Every material changes shape to some extent when a force is applied to it. This is sometimes difficult to see in materials such concrete and we need special equipment to detect these changes.

If a compressive load is applied to a structural member, then the length will reduce. If a tensile load is applied, then the length will increase. This is shown in the diagrams below.



The result of applying a load to a structural member is called STRAIN. This is calculated using the formula:

$$Strain = \frac{Change\ in\ Length}{Original\ Length}$$

$$\varepsilon = \frac{\Delta L}{L}$$

where length in both cases is measured in the same units (m or mm). As the units cancel each other out, strain is dimensionless. This means that there are no units of strain. Put simply, strain is a ratio that describes the proportional change in length in the structural member when a direct load is applied. Strain is denoted by the Greek letter epsilon (ε).

Worked examples: Strain

1. A steel wire of length 5 m is used to support a tensile load. When the load is applied, the wire is found to have stretched by 2.5 mm. Calculate the strain for the wire.

$$\varepsilon = \frac{\Delta L}{L}$$

$$\varepsilon = \frac{2.5}{5000}$$

$$\varepsilon = 0.0005$$

Strain in the wire = 0.0005

2. The strain in a concrete column must not exceed 5 x 10-4. If the column is 3 m high, find the maximum reduction in length produced when the column is loaded.

$$\varepsilon = \frac{\Delta L}{L}$$

$$\Delta L = \varepsilon \times L$$

$$\Delta L = \left(5 \times 10^{-4}\right) \times 3000$$

$$\Delta L = 1.5mm$$

Reduction in length of column = 1.5 mm

Assignments: Stress and Strain

1. A bar of steel 500 mm long has a cross-sectional area of 250 mm2 and is subjected to a force of 50 kN. Determine the stress in the bar.

2. A wire 4 mm in diameter is subjected to a force of 300 N. Find the stress in the wire.

3. What diameter of round steel bar is required to carry a tensile force of 10 kN if the stress is not to exceed 14.14 N/mm2.

4. A wire 10 m long stretches 5 mm when a force is applied at one end. Find the strain produced.

5. A tow bar, 1.5 m long, is compressed by 4.5 mm during braking. Find the strain.

6. The allowable strain on a bar is 0.0075 and its length is 2.5 m. Find the change in length.

7. During testing, a shaft was compressed by 0.06 mm. If the resulting strain was 0.00012, what was the original length of the shaft?

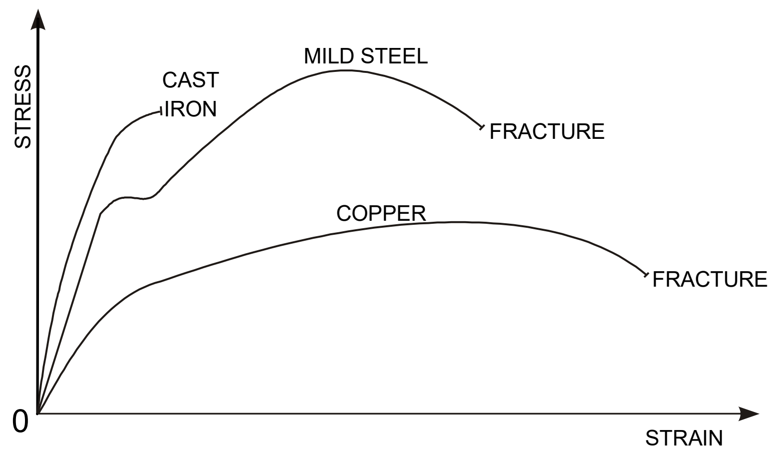
8. A piece of wire 6 m long and diameter of 0.75 mm stretched 24 mm under a force of 120 N. Calculate stress and strain.

9. A mild steel tie-bar, of circular cross-section, lengthens 1.5 mm under a steady pull of 75 kN. The original dimensions of the bar were 5 m long and 30 mm in diameter. Find the intensity of tensile stress in the bar and determine the strain.

10. A mass of 2500 kg is hung at the end of a vertical bar, the cross-section of which is 75 mm x 50 mm. A change in length in the bar is detected and found to be 2.5 mm. If the original length of the bar is 0.5 m, calculate the stress and strain in the bar.

Using Data from Stress Strain Graphs

As we have already learned, vital information can be obtained from tensile tests when the data is plotted in the form of a stress strain graph. The graph below represents the relationship between stress and strain for common materials.



The following points are important in relation to the graph.

1. Yield Stress

The yield stress is the maximum stress that can be applied to a structural member without causing a permanent change in length. The loading on any structural member should never produce a stress that is greater than the yield stress. That is, the material should remain elastic under loading.

2. Yield Strain

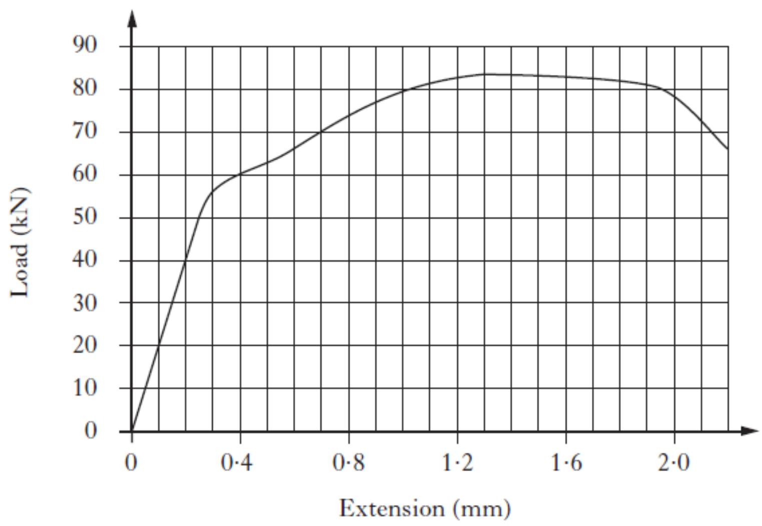
The yield strain is the maximum percentage plastic extension produced in a material before it fails under loading. A ductile material such as copper needs to be formed and shaped into items such as pipes. For this to be effective, the material requires a high value of yield strain.

3. Ultimate Tensile Stress

The ultimate tensile stress (UTS) of a material is the maximum stress the material can withstand before it starts to fail. If a member in a structure is loaded beyond the UTS, the cross-section will reduce and the member will quickly fail.

Questions

1. Below is the load-extension graph produced during a tensile test performed on an alloy-steel specimen.



The test specimen was 120mm long with a rectangular cross-section of 26mm x 6mm.

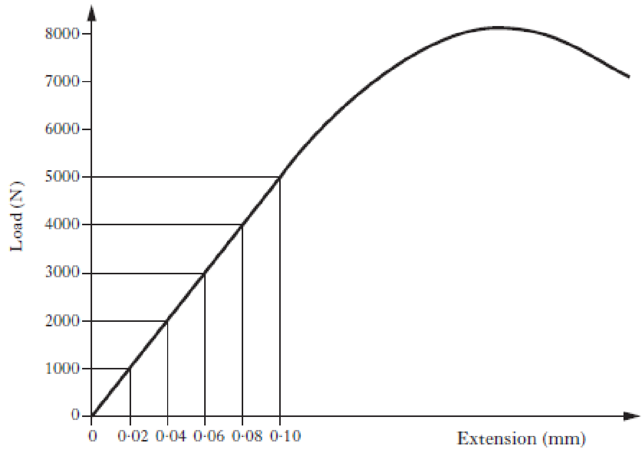
a) Calculate Young's Modulus for this material.

b) Describe the effect on the specimen of applying and then removing the following loads:

   i) 50kN

   ii) 80kN

2. A load-extension graph for a standard test specimen is shown below. The specimen is 200mm long and 11.3mm in diameter.



a) Calculate Young's Modulus for the test specimen.

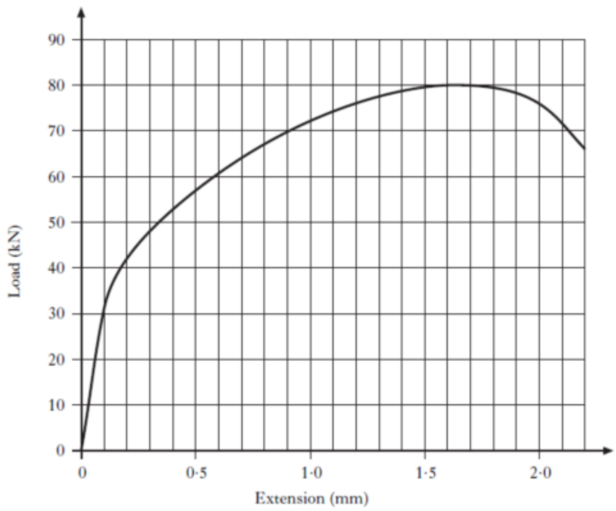
b) State the name of the material from which the test specimen is made.

c) Sketch the above graph, and on it show the yield point, the ultimate load, the plastic range and the elastic range.

Two further specimens were tensile tested. The dominant mechanical property of specimen A was brittleness, and that of B was ductility.

d) Sketch, on the same axes, typical stress-strain graphs for specimen A and specimen B. Clearly label the axes and identify each graph.

3. A load-extension graph is shown below for a tensile test on a sample of a special type of steel.



a) State whether this material is brittle or ductile.

The test specimen was 50mm long with a cross-sectional area of 80mm^2.

b) Calculate Young's Modulus for the material.

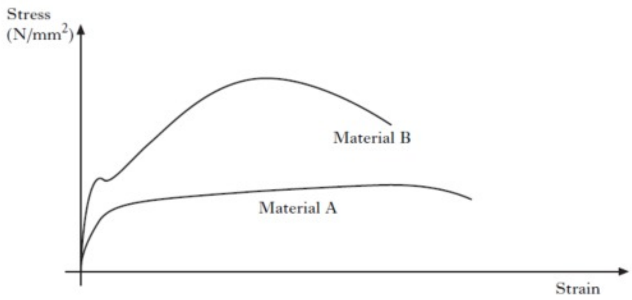
c) Calculate the ultimate tensile stress for the material.

A bar with a rectangular cross-section of 30mm x 10mm is manufactured from the material. The bar is to be loaded in tension with a Factor of Safety of 6.

d) i) Calculate the safe working stress for the bar.

ii) Calculate the safe working load for the bar.

4. The stress-strain graphs for two different materials are shown below.



a) From the information given, state one property of material A, giving a reason for your choice.

b) Describe the difference between elastic and plastic deformation.

Another material with a gauge length of 120mm and a cross-sectional area of 12mm^2 was tested. It was found that at just below the yield point a force of 2.96kN caused an increase in length of 0.185mm.

c) i) Calculate the Modulus of Elasticity for this material.

ii) State the name of the material.

iii) State one reason why this material might be selected for a cable supporting an overhead lighting gantry.