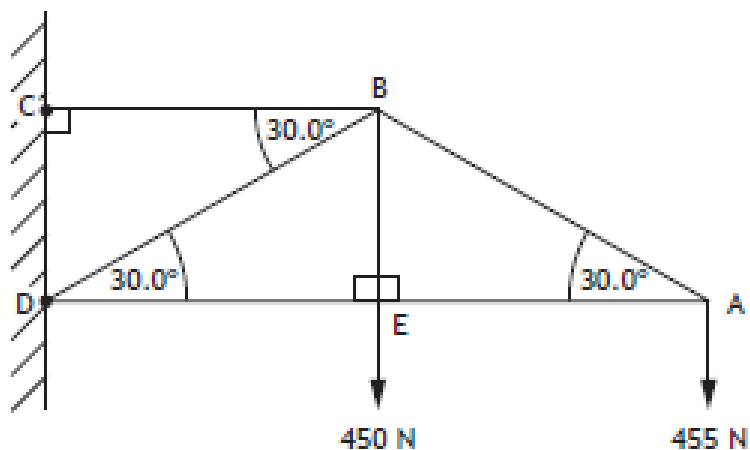


Higher Engineering Science

Structures

Including UDLs, Nodal Analysis &
Simultaneous Equations



Name: _____

Teacher: _____

Class: _____



Mrs Gault

Learning Intentions

- Resolving triangle/polygon of forces
- Resolving resultant forces for systems in equilibrium
- Solving reaction forces in simply supported beams:
 - where loads are not exclusively vertical or horizontal
 - with hinge and roller supports
 - With uniformly distributed loads
- Nodal analysis of a frame
- Simultaneous equations of structures

Success Criteria

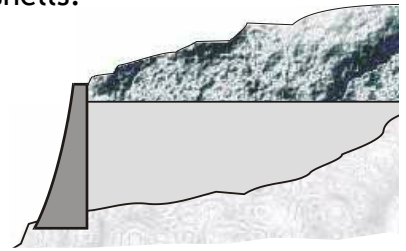
- I can resolve forces at an angle into their vertical and horizontal components using Pythagoras.
- I can find the unknown forces in a system by using one of 3 equations.
- I can solve equations with loads at an angle, hinge and roller supports and UDLs.
- I can complete a nodal analysis question to work out internal forces and their nature.
- I can complete simultaneous equations using my knowledge of Pythagoras and the 3 equations for equilibrium.

In this unit we will build on the skills learned in National 5 Engineering Science.

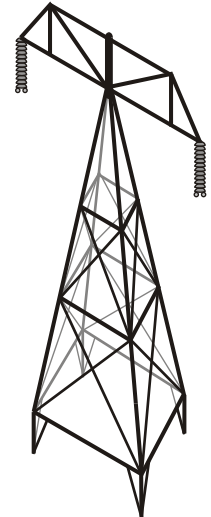
STRUCTURES

There are three main types of structure - mass, framed and shells.

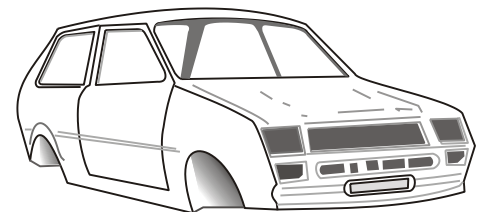
Mass structures perform due to their own weight.
An example would be a dam.



Frame structures resist loads due to the arrangement of its members.
A house roof truss can support a load many times its own weight.
Electricity pylons are good examples of frame structures.



Shells are structures where its strength comes from the formation of sheets to give strength. A car body is an example of a shell structure.



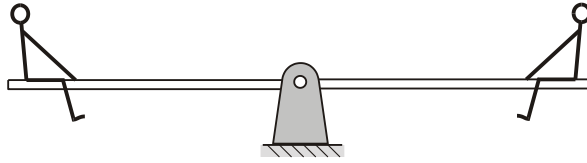
It is very important with any structure that we can calculate the forces acting within it so that a safe structure can be designed. Early structures were found to be successful due to the fact that they stayed up and many early structures are still with us, but many are not. The science of structures has been progressively improving over the centuries and it is now possible to predict structures behaviour by analysis and calculation. Errors can still be made, sometimes with catastrophic results.

These are the 3 equations we will use throughout this topic.

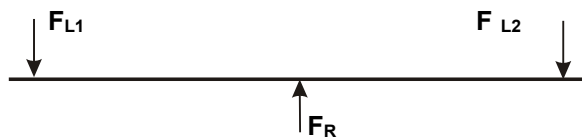
Vertical Forces $\Sigma F \uparrow = \Sigma F \downarrow$	Horizontal Forces $\Sigma F \rightarrow = \Sigma F \leftarrow$	Moments $\Sigma CW M = \Sigma AC W M$
---------------------------------------------------------------------	--------------------------------------------------------------------------	-------------------------------------------------

FORCES

In the structure below, three forces are acting on it.



The diagram shown below can represent the forces in the above diagram.



F_{L1} and F_{L2} represent the force exerted due to the mass of the people. F_R is the reaction force. This type of diagram is known as a **free-body diagram**.

The reaction load F_R is found by adding the two downward forces together.

In any force system the sum of the vertical forces must be equal to zero.

$$\Sigma F_{V\uparrow} = \Sigma F_{V\downarrow}$$

$$F_R = F_{L1} + F_{L2}$$

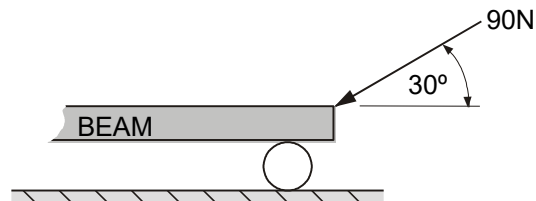
$$F_R = 810\text{N} + 740\text{N}$$

$$F_R = 1550\text{N}$$

RESOLUTION OF A FORCE

In some cases the loads may not be acting in the same direction, and cannot therefore be added together directly.

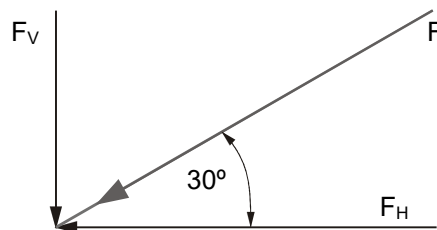
In the situation shown below the force is acting down at an angle.



This force can be split into two separate components:

A vertical component F_V .

A horizontal component F_H .



To resolve a force into its components you will have to know two things, its magnitude and direction.

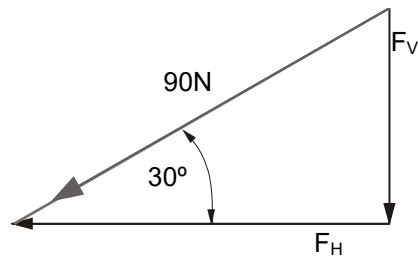
Trigonometry is used to resolve forces.

Where - Hypotenuse = force, F

Opposite = vertical component, F_V

Adjacent = horizontal component, F_H

The diagram above can be redrawn as below.



To find the horizontal force, F_H

$$\begin{aligned}\cos 30^\circ &= \frac{F_H}{90} \\ F_H &= 90 \times \cos 30^\circ \\ &= 77.94N\end{aligned}$$

Horizontal force = 77.94 N

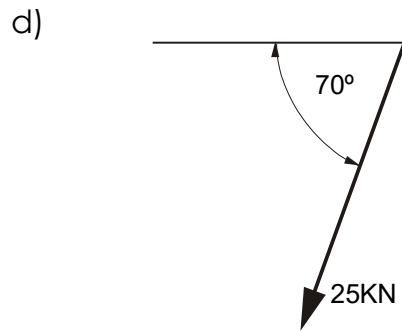
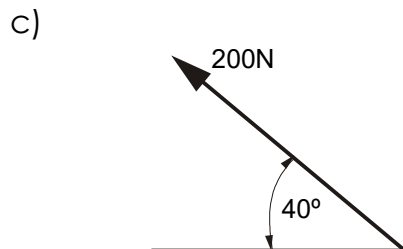
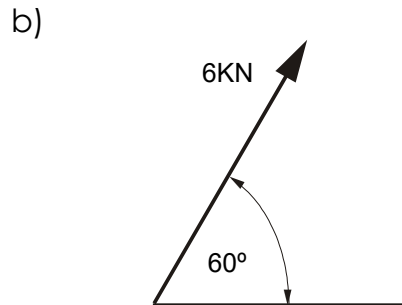
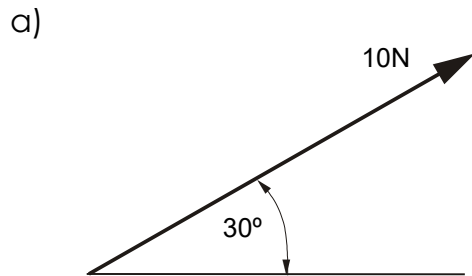
To find the vertical force, F_V

$$\begin{aligned}\sin 30^\circ &= \frac{F_V}{90} \\ F_V &= 90 \times \sin 30^\circ \\ &= 45N\end{aligned}$$

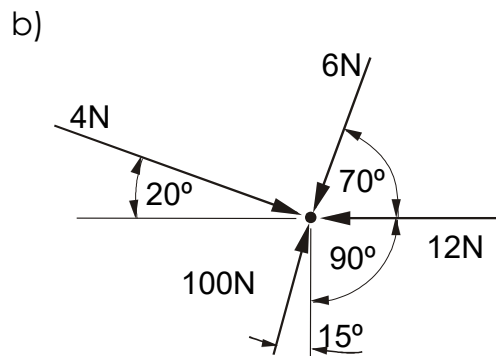
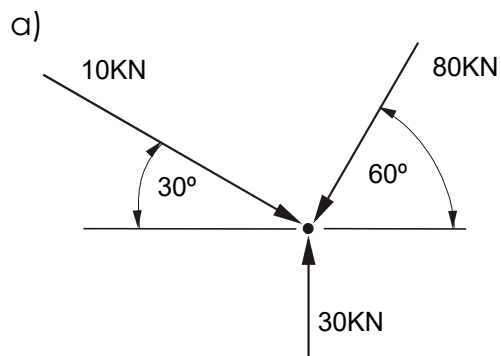
Vertical force = 45 N

Task - Resolution of Forces

1. Resolve the following forces into their horizontal and vertical components.

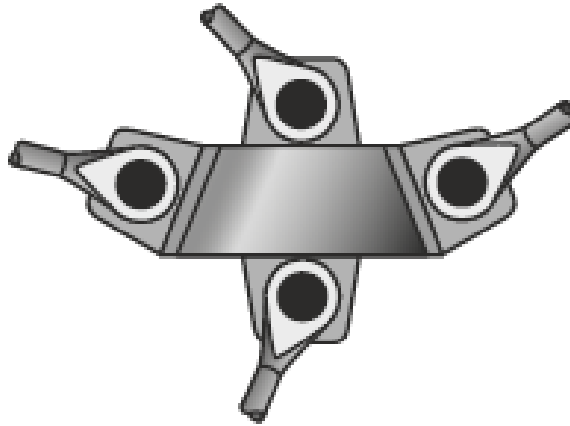


2. Find the resultant for these force systems. Find the horizontal and vertical components of each - add them up to find the overall component forces and then find the resultant.

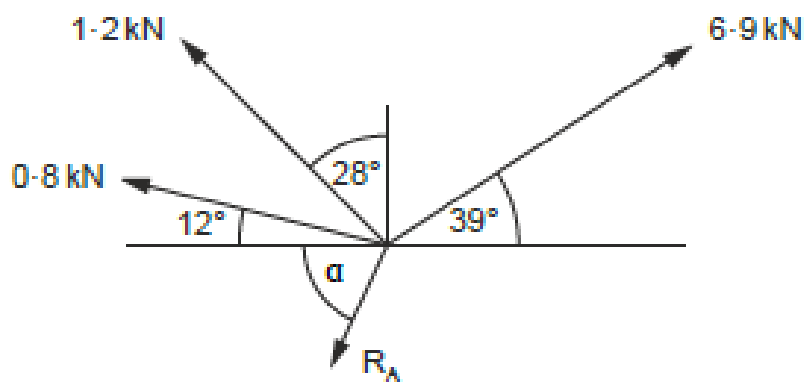


Task 1

The joint shown in below forms part of the roof structure of a new museum.



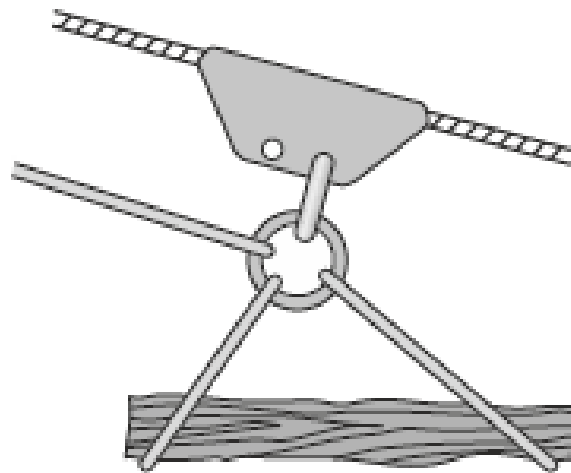
The forces acting on the joint are in static equilibrium and are shown *in simplified form* below.



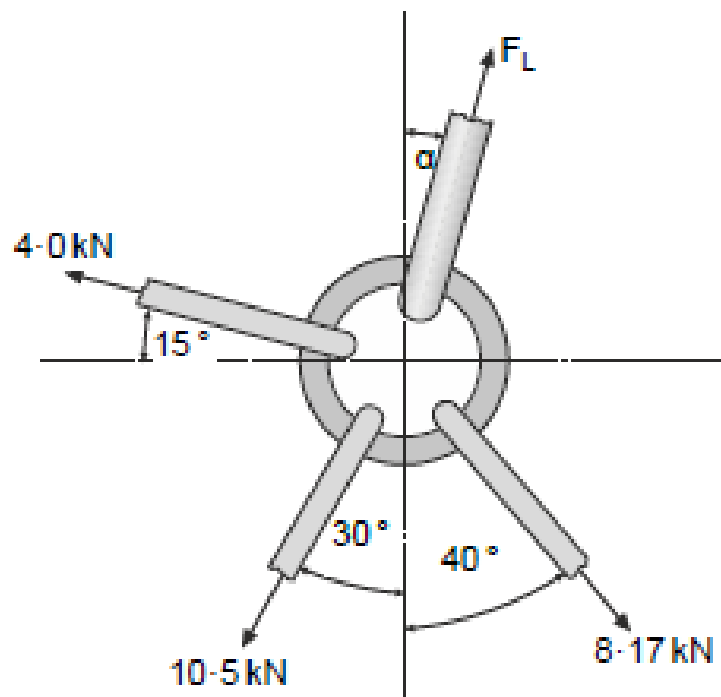
Calculate the magnitude of the reaction force R_A and the angle α .

Task 2

A tree-harvesting operation transports cut logs with a system similar to that shown below.



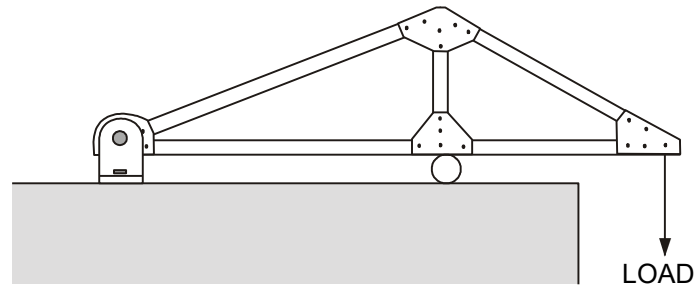
The forces acting on the connecting ring are also shown. All forces act concurrently.



For the forces shown, calculate the magnitude and direction of force F_L in the link.

MOMENT OF A FORCE

The moment of a force is the turning effect of that force when it acts on a body

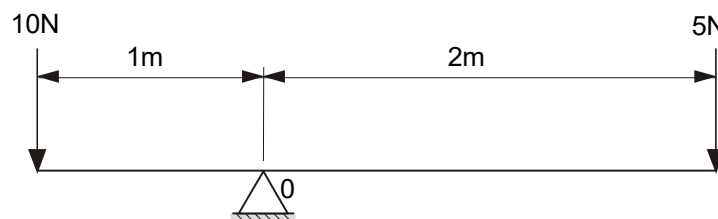


The load acting on the frame structure above will have a turning effect on the structure.

The Principle of Moments states that if a body is in Equilibrium the sum of the clockwise moments is equal to the sum of anti-clockwise moments.

Worked Examples

Example 1



$$\sum CwM = \sum ACwM$$

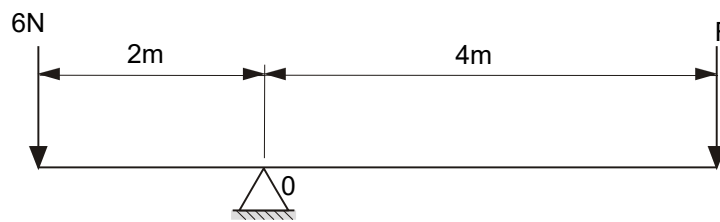
$$F \times d = F \times d$$

$$5 \times 2 = 10 \times 1$$

The above example is in equilibrium.

We can use this principle to find an unknown force or unknown distance.

Example 2



$$CwM = ACwM$$

$$(F \times 4) = (6 \times 2)$$

$$4F = 12$$

$$F = 12/4$$

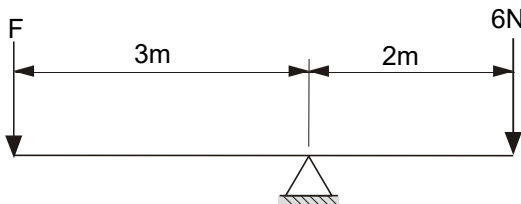
$$F = 3N$$

Assignments: Moments

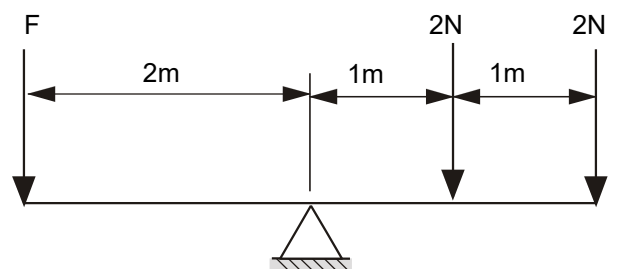
The beams shown below are in equilibrium. Find the unknown quantity for each arrangement.

Question 1

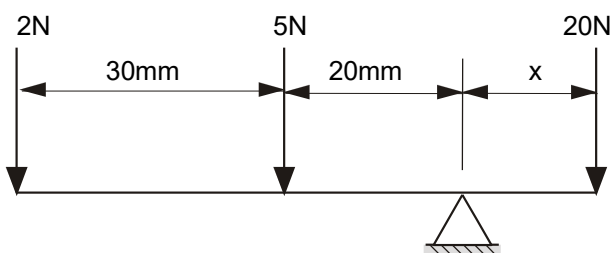
a)



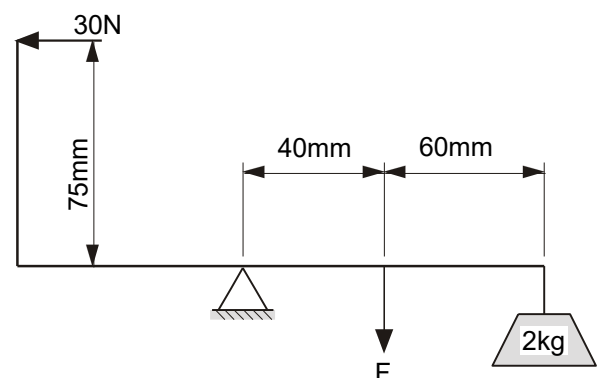
b)



c)



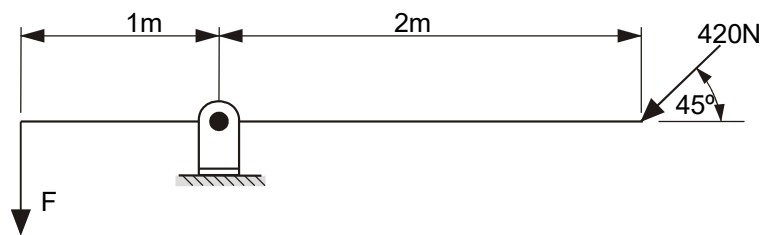
d)



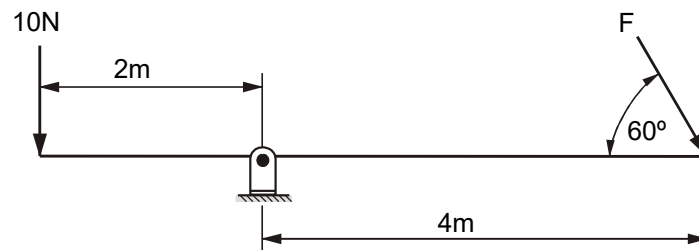
The following beams, in equilibrium, have inclined forces. Find the unknown quantity.

Question 2

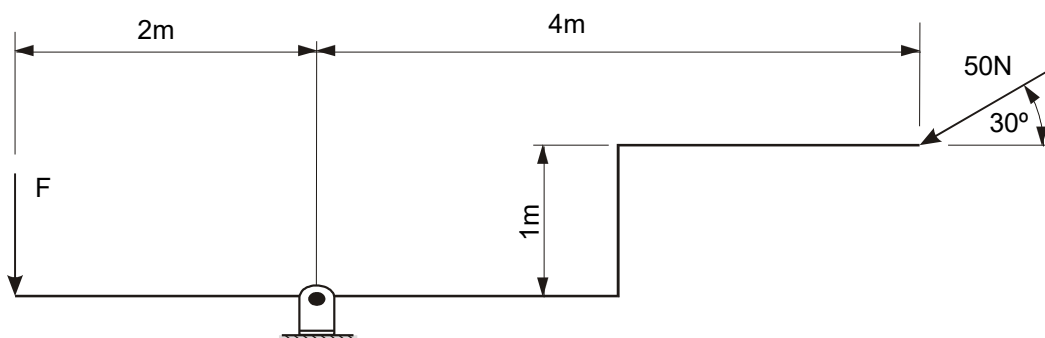
a)



b)



c)



BEAM REACTIONS

We are now going to study beams with external forces acting on them. We shall resolve forces into their components and use moments to find the support reactions.

Definitions of some of the terms you have met already:

RESULTANT The resultant is that single force that replaces a system of forces.

EQUILIBRIUM Equilibrium is the word used to mean balanced forces.

Conditions of Equilibrium

1. The sum of the Clockwise Moments = the sum of the Anti-Clockwise Moments

$$\Sigma CWM = \Sigma ACWM$$

2. The sum of the forces acting upwards equals the sum of the forces acting downwards.

$$\Sigma F \uparrow = \Sigma F \downarrow$$

3. The sum of the forces acting to the right equal the sum of the forces acting to the left.

$$\Sigma F \rightarrow = \Sigma F \leftarrow$$

Beam Reactions

A beam is usually supported at two points. There are two main ways of supporting a beam -

1. Simple supports (knife edge)



2. Hinge and roller

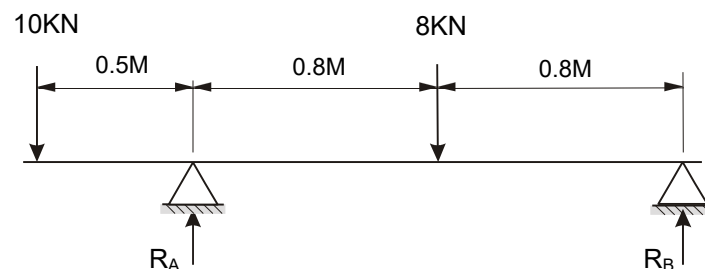


Worked Examples

1. SIMPLE SUPPORTS

Simple supports are used when there is no sideways tendency to move the beam.

Consider this loaded beam, “simply” supported.



1. The forces at the supports called reactions, always act vertically.

2. The beam is in equilibrium; therefore the conditions of equilibrium apply.

The value of Reactions R_A and R_B are found as follows.

Take moments about R_A

$$CWM = ACWM$$

$$(8 \times 0.8) = (R_b \times 1.6) + (10 \times 0.5)$$

$$6.4 = 1.6R_b + 5$$

$$1.6R_b = 6.4 - 5$$

$$R_b = 1.4 / 1.6$$

$$R_b = 0.88\text{kN}$$

To find R_A

$$\sum F\uparrow = \sum F\downarrow$$

$$R_a + R_b = 10 + 8$$

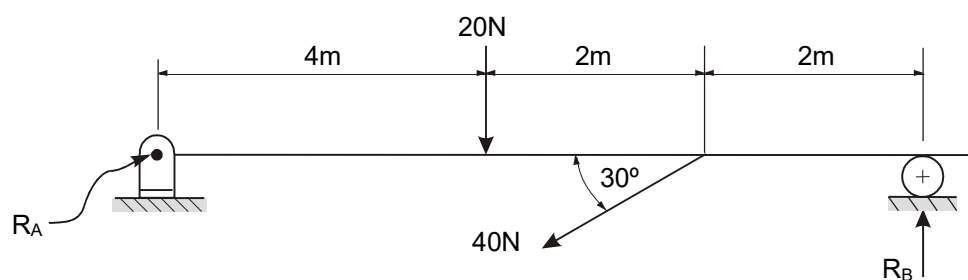
$$R_a = 18 - R_b$$

$$R_a = 18 - 0.88$$

$$R_a = 17.12\text{kN}$$

2. HINGE AND ROLLER SUPPORTS

Hinge and roller supports are used when there is a possibility that the beam may move sideways.



Note: The reaction at a roller support is always at right angles to the surface. The direction of R_A is assumed. If any of the components work out as negative values then the direction will be opposite the assumed direction.

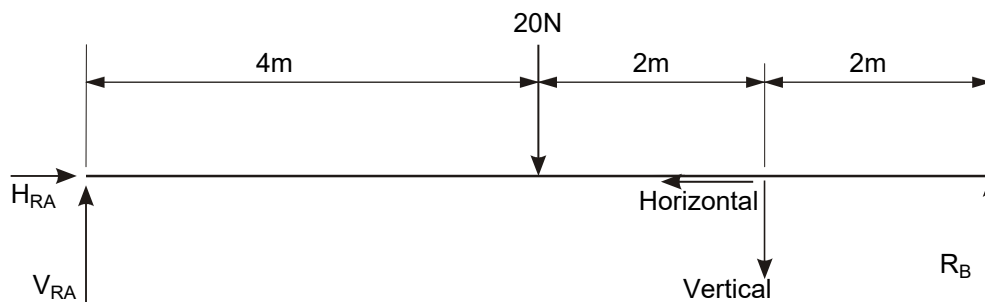
The reaction at the hinge support can be any direction.

(Find the two components of the hinge reaction, then the resultant)

There are three unknown quantities above: -

1. The magnitude of Reaction R_B .
2. The magnitude of Reaction R_A .
3. The direction of Reaction R_A .

Redraw as a free-body diagram showing vertical and horizontal components of the forces.



The vertical and horizontal components of the 40N force are found.

$$V = \sin 30^\circ \times 40$$

$$H = \cos 30^\circ \times 40$$

$$V = 20N$$

$$H = 34.64N$$

To find R_B -take moments about V_{RA} , this eliminates one of the unknown vertical forces.

$$CWM = ACWM$$

$$20 \times 4 + 20 \times 6 = R_B \times 8$$

$$8R_B = 200$$

$$R_B = \frac{200}{8}$$

$$R_B = 25N$$

Vertical forces

$$\sum F_{\uparrow} = \sum F_{\downarrow}$$

$$V_{RA} + R_B = 20 + 20$$

$$V_{RA} = 40 - R_B$$

$$V_{RA} = 40 - 25$$

$$V_{RA} = 15N$$

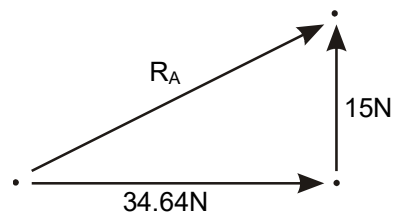
Horizontal forces

$$\sum F_{\rightarrow} = \sum F_{\leftarrow}$$

$$H_{RA} - 34.64 = 0$$

$$H_{RA} = 34.64N$$

Use V_{RA} and H_{RA} to find R_A



$$R_A = \sqrt{15^2 + 34.64^2}$$

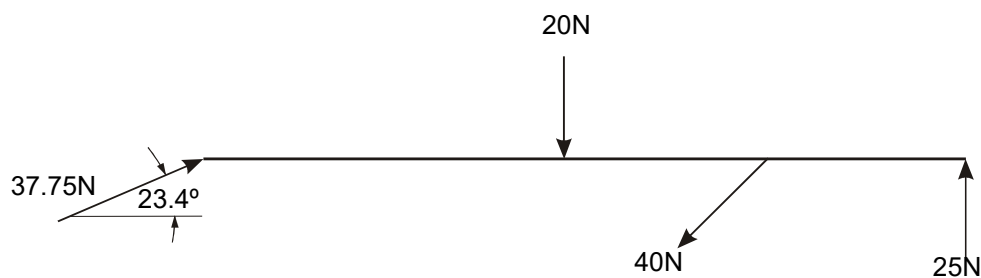
$$R_A = 37.75N$$

Find direction of R_A

$$\tan \theta = \frac{15}{34.64}$$

$$\theta = \tan^{-1}\left(\frac{15}{34.64}\right)$$

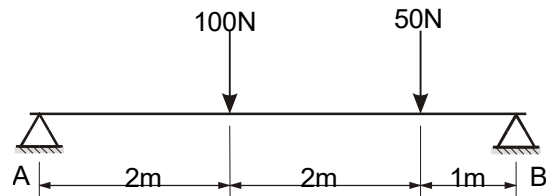
$$\theta = 23.4^\circ$$



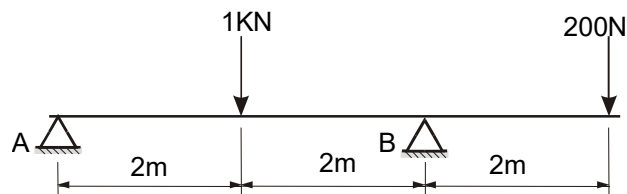
Assignments: Beam Reaction

1 Find the reactions at supports A and B for each of the loaded beams shown below.

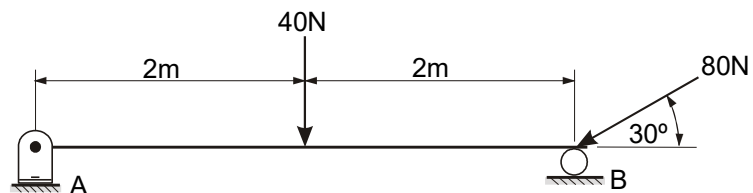
a)



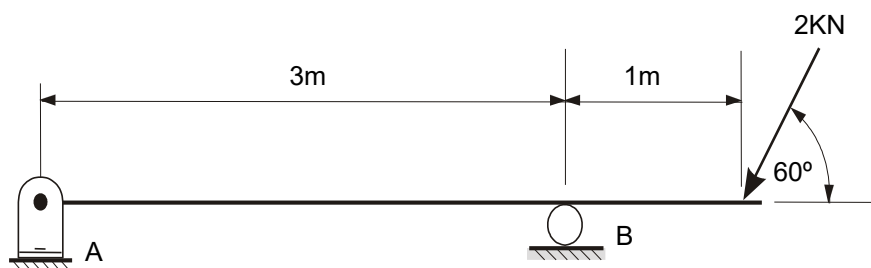
b)



c)



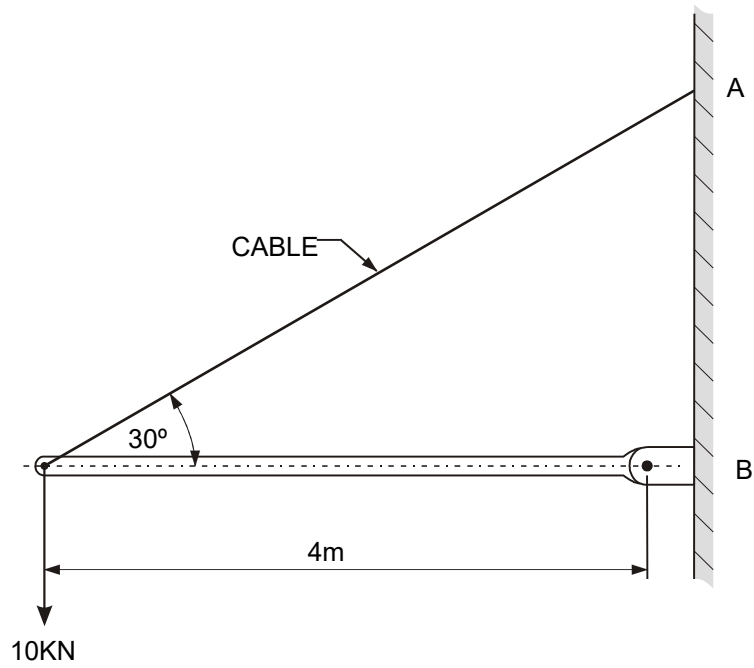
d)



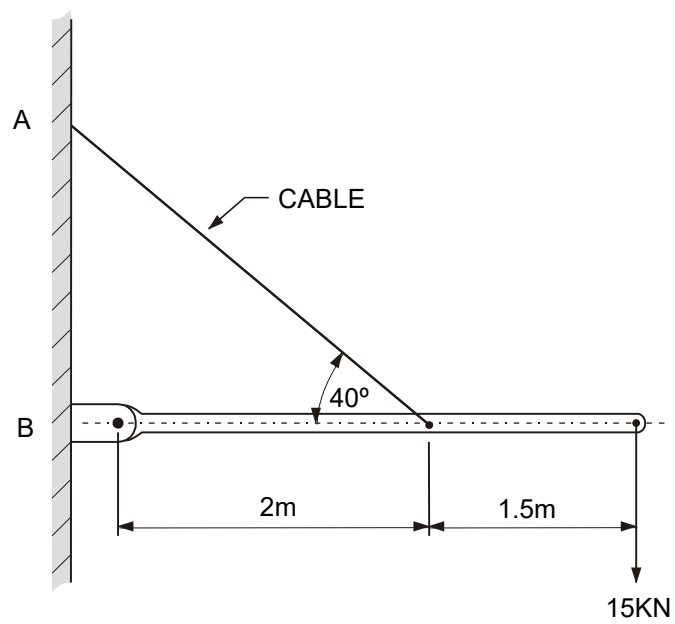
Task - Beam Reaction (continued)

2) Find the reactions at supports A and B for each of the loaded beams shown below.

a)

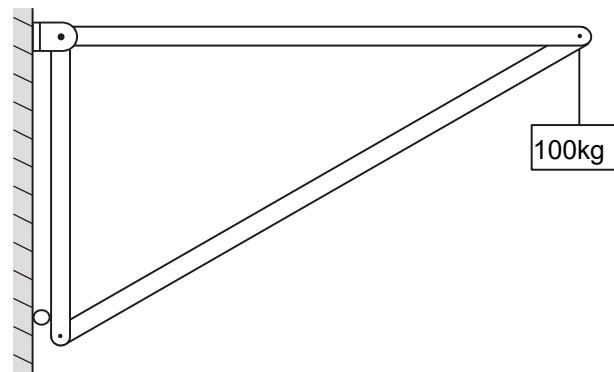


b)



FRAMED STRUCTURES

A frame structure is an assembly of members and joints (usually called Nodes) which is designed to support a load. Examples of frame structures include roof trusses, bridges, pylon towers.



The members in this framed structure can be as ties or struts, depending on the type of force they support.

STRUTS AND TIES

When solving problems in frame structures you will be required to determine the Magnitude and Nature of the forces in the members of the frame. That is, determine, in addition to the size of the force in the member, whether the member is a Strut or a Tie.

Strut

Members that are in compression, due to external forces trying to compress them, are known as Struts.



Tie

Members that are in tension, due to external forces trying to pull them apart, are known as Ties.



Nodal Analysis

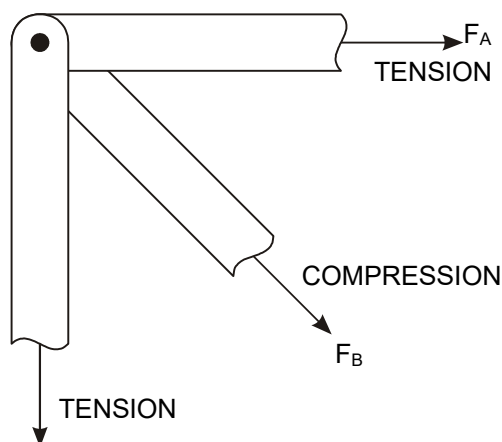
There are several methods of solving frame structure problems. The method we shall use is called Nodal Analysis.

Any joint where members meet is known as a node.

This method relies on the fact that if structures are in equilibrium then each node will be in equilibrium. The sum of the forces acting on any the node will equal zero.

Nodes

The members are either in compression (strut) or tension (tie). They can be represented at the node as shown below -



F_B is under compression and pushes into the node, F_A is under tension and pulls away from the node.

Conditions of Static Equilibrium

SUM OF THE MOMENTS = 0	$(\sum M_o = 0)$	$\sum CWM = \sum ACWM$
SUM OF THE VERTICAL FORCES = 0	$(\sum F_v = 0)$	$\sum F\uparrow = \sum F\downarrow$
SUM OF THE HORIZONTAL FORCES = 0	$(\sum F_H = 0)$	$\sum F\rightarrow = \sum F\leftarrow$

When solving frame structures some analysis of the structure is required to determine the best starting point and which of the conditions of static equilibrium to apply first.

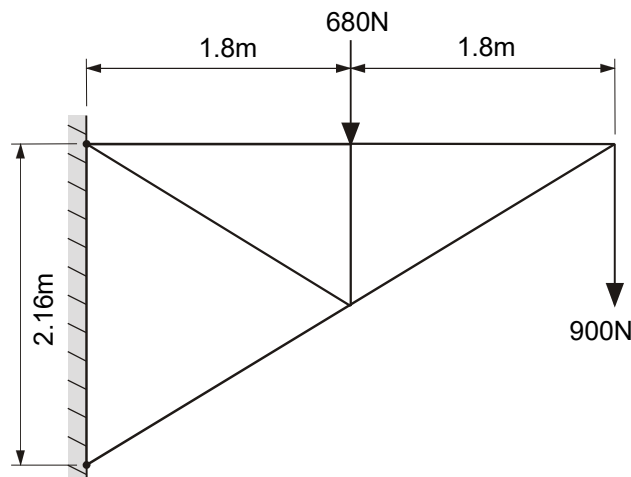
Solving Simple Frame Structures

To help solve frame structure problems there are some simple rules to follow depending on the type of structure.

Cantilevered frame structures

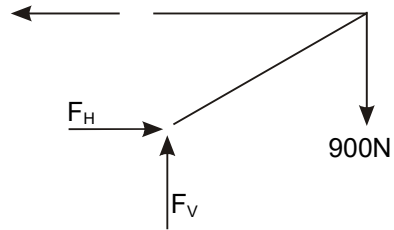
For this frame structure it is not necessary to use moments to help find the forces in the members. .

The node with the 900N load acting has only one unknown vertical component.



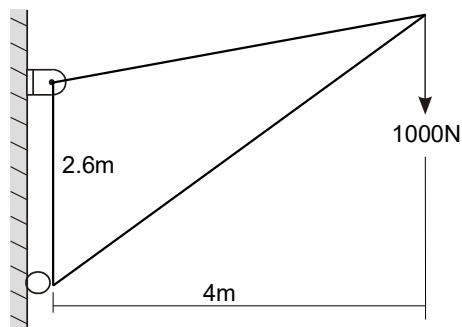
As all vertical forces acting on the node must equal zero then F_v must equal 900N.

By using trigonometry it is now possible to find the other forces acting on this node.

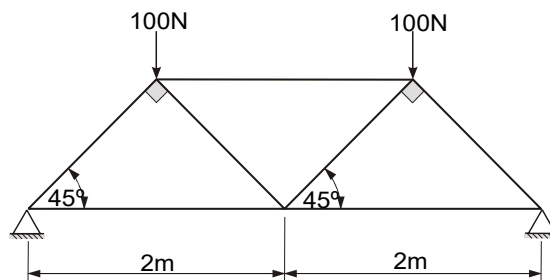


In this example all nodes have more than one unknown force or component of a force.

To solve this frame structure take moments about the top support to find the reaction force at the roller support.



Truss frame structures

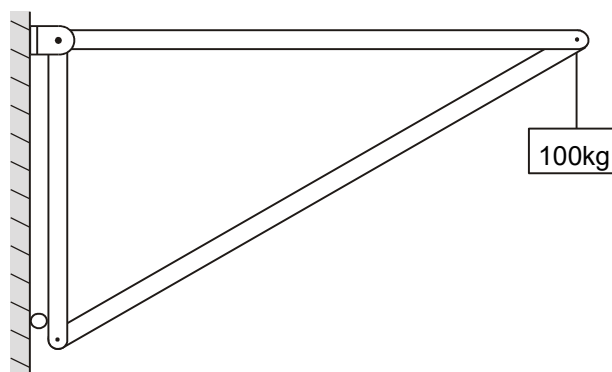


Treat structure like this as a beam and use moments to find reactions. Until the reaction forces are found all nodes have more than one unknown force or component.

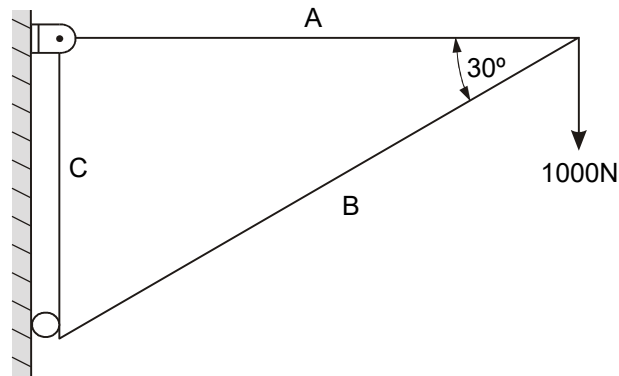
Then start analysis at R_1 or R_2 , which will now have only one unknown component.

Worked Example

For the crane shown below we shall find the forces in the members and the reaction forces at the wall.



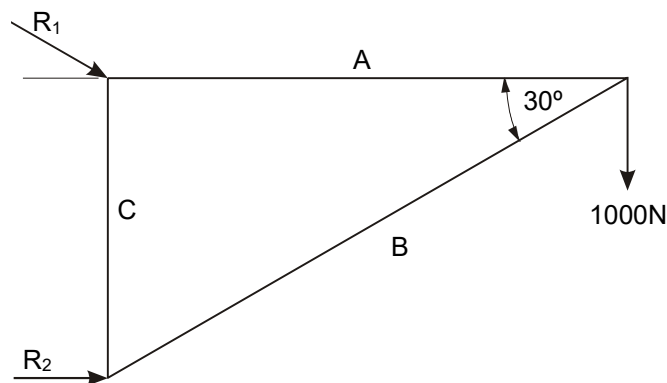
Draw a free body diagram of the frame structure.



The framework is supported at two points. The hinge support at the top is being pulled away from the wall. R_1 will act against this pull and keep the hinge attached to the wall. The roller support is being pushed into the wall. R_2 will act against this force and in the opposite direction as shown below.

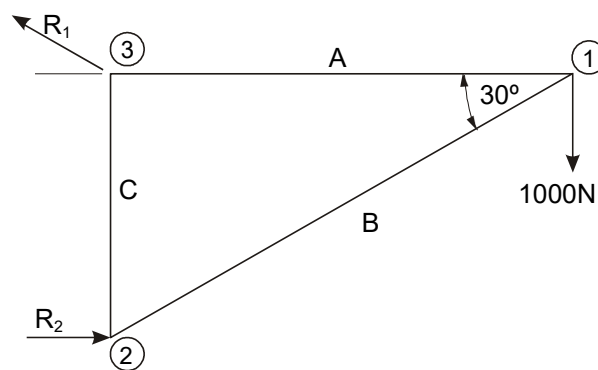
As member **B** is acting on a roller then R_2 will be at 90° to member **C**

As R_1 is acting at a hinge support there will be a vertical and horizontal component. At this stage guess the direction of R_1 .



If a wrong assumption about the member being in tension or compression or the direction of a force is made then a negative value will be produced from the calculation. If this is the case the direction of the force is simply reversed

To find the forces in the members and the reactions at the supports, study each of the structure nodes separately.



Each node is in equilibrium so we can use the conditions of equilibrium.

SUM OF THE VERTICAL FORCES = 0

$$\sum F_v = 0 \quad \sum CWM = \sum ACWM$$

SUM OF THE HORIZONTAL FORCES = 0

$$\sum F_H = 0 \quad \sum F\uparrow = \sum F\downarrow$$

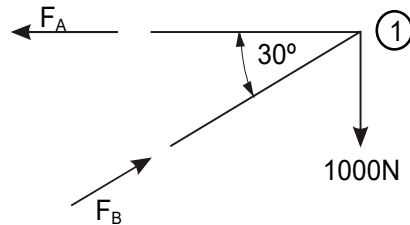
SUM OF THE MOMENTS = 0

$$\sum M_o = 0 \quad \sum F\rightarrow = \sum F\leftarrow$$

In this sort of problem where the structure is cantilevered out from the wall, it is normally solvable without considering the moments. Start by finding a node that has only one unknown vertical or horizontal component.

Node 1

The forces acting on node 1 are shown below.

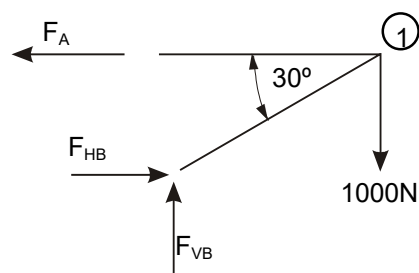


Note - an assumption has been made about the direction of F_A and F_B .

F_A is assumed to be in tension and acting away from the node 1.

F_B is assumed to be in compression and acting in towards the node 1.

Split F_B into its horizontal (F_{HB}) and vertical (F_{VB}) components.

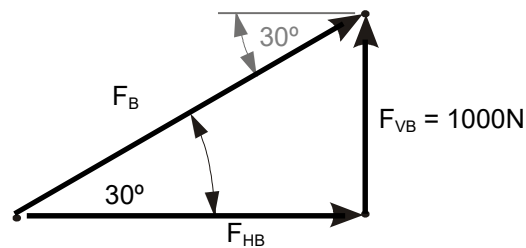


Apply a condition of static equilibrium -

$$\sum F_v = 0$$

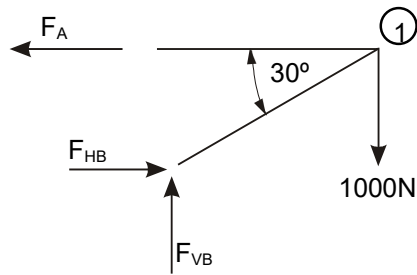
As the sum of Vertical forces is equal to zero then the vertical component of F_B (F_{VB}) is 1000N acting up.

From this we can find F_B - we can redraw F_B and the two components F_{HB} and F_{VB} to form a triangle as shown below.

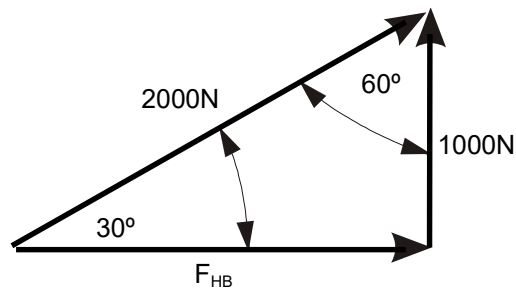


$$\begin{aligned}\sin 30^\circ &= \frac{F_{VB}}{F_B} \\ F_B &= \frac{F_{VB}}{\sin 30^\circ} \\ &= \frac{1000}{0.5} \\ &= 2000N\end{aligned}$$

The force in member A (F_A) will equal and opposite to F_{HB} .



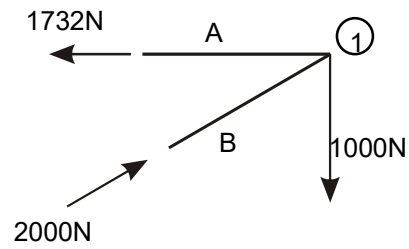
Find F_{HB}



$$\begin{aligned} \cos 30^\circ &= \frac{F_{HB}}{F_B} \\ F_{HB} &= \cos 30^\circ \times F_B \\ &= 0.87 \times 2000 \\ &= 1732N \end{aligned}$$

From above $F_A = 1732N$

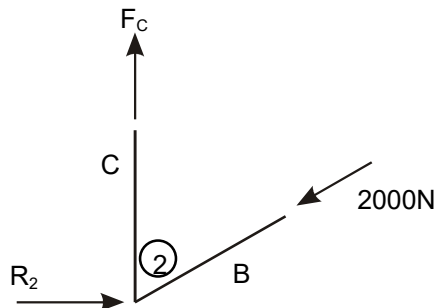
This diagram shows all the forces acting at node 1.



Worked Example (continued)

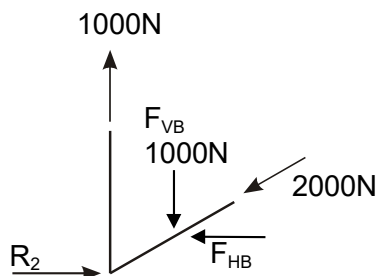
Node 2

There are three forces acting at node 2.



During analysis of node 1, F_B was found to be 2000N and was not negative so the assumed direction towards the node (compression) was correct.

In node 1, F_B is shown to be acting as a compressive force towards the node with a magnitude of 2000N.



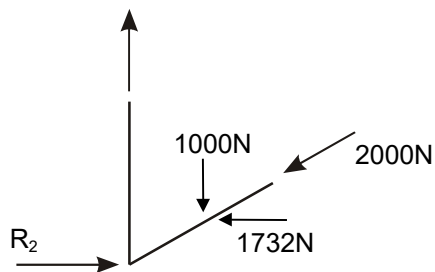
F_c will be equal and opposite to the Vertical component of F_B (F_{VB}).

From analysis of node 1 F_{VB} was equal to 1000 N therefore -

$$F_C = 1000 \text{ N}$$

As R_2 is a roller support the reaction will be at 90° to the surface.

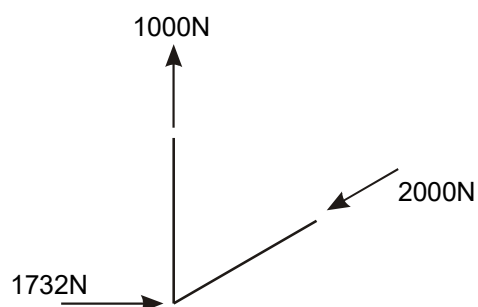
R_2 will be equal to the Horizontal component of F_B .



From above H_B was equal to 1732 N therefore -

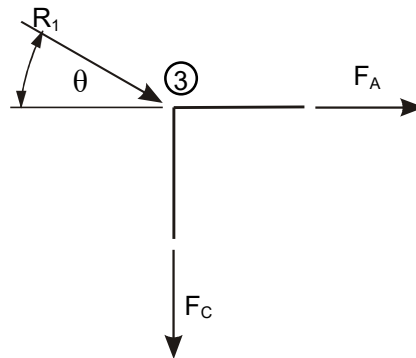
$$R_2 = 1732 \text{ N}$$

The forces acting at node 2 are shown below.



Node 3

Node 3 has three forces acting on it.

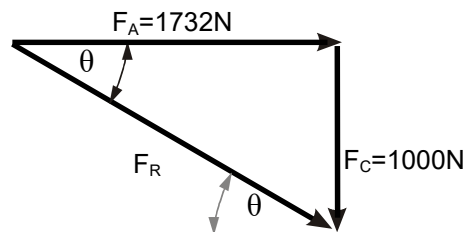


R_1 will be equal and opposite to the resultant of F_A and F_C .

From analysis of node 1 F_A was found to be 1732N.

From analysis of node 2 F_C was found to be 1000N

Find resultant of F_A & F_C - F_R



$$F_R = \sqrt{1732^2 + 1000^2}$$

$$F_R = 2000N$$

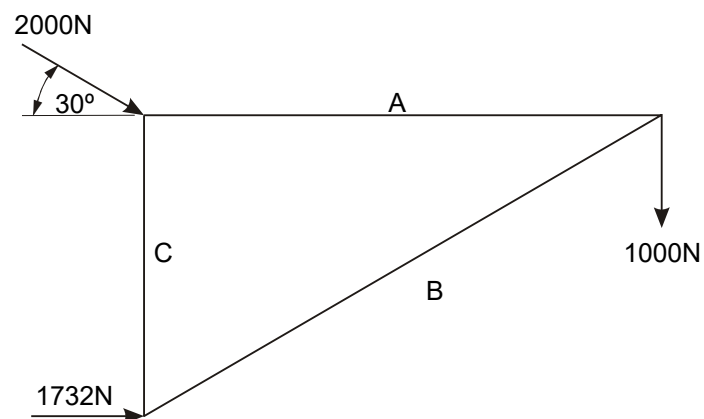
Find angle of resultant (θ) -

$$\tan\theta = \frac{1000}{1732}$$

$$\theta = \tan^{-1}\left(\frac{1000}{1732}\right)$$

$$\theta = 30^\circ$$

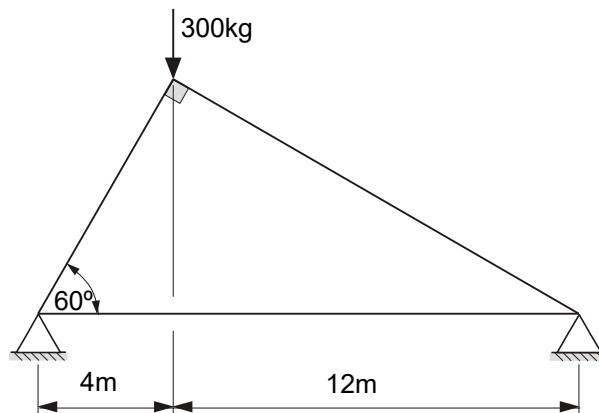
The task was to find the forces in the members and the reactions forces at the supports. The results can be shown as below -



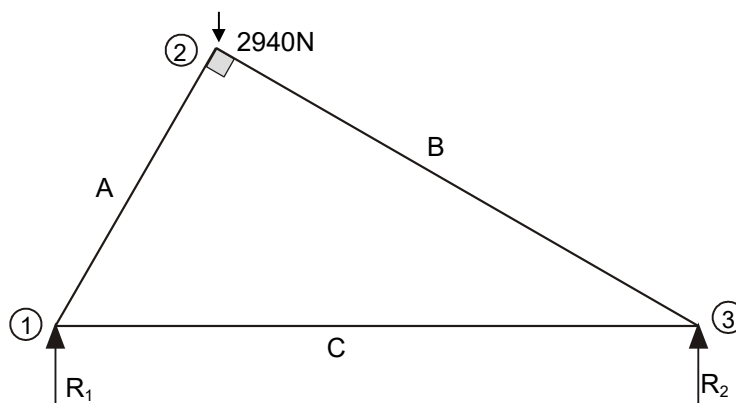
MEMBER	FORCE	TYPE OF FORCE
A	1732N	TENSION
B	2000N	COMPRESSION
C	1000N	TENSION

Assignment 1

Find the reactions R_1 and R_2 and the forces for the members in the frame structure shown below.



The free body diagram with the nodes and members labelled is given below.



In this type of problem where the structure is acting as a beam, find R_1 and R_2 using moments.

Take moments about R_1 -

$$\Sigma CW M = \Sigma ACW M$$

$$(2940 \times 4) = R_2 \times 16$$

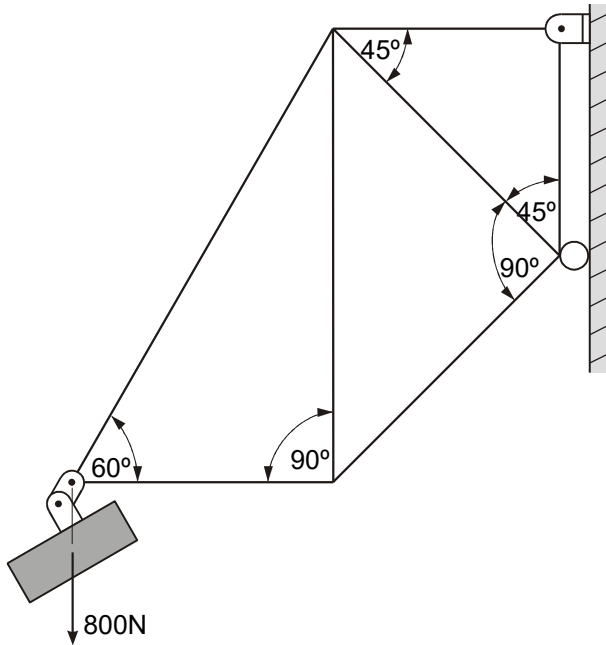
$$R_2 = \frac{11760}{16}$$

$$R_2 = 735N$$

Now complete this task and find R_1 and the forces in A, B and C.

Assignment 3

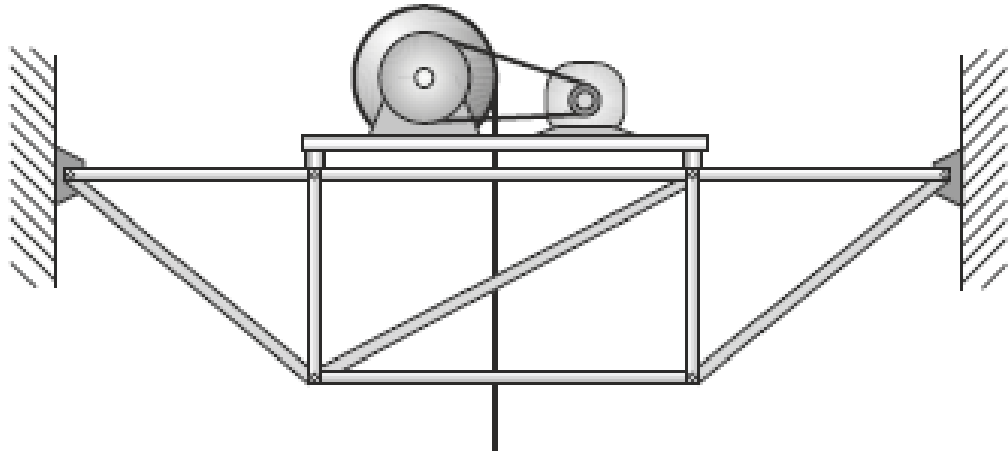
Calculate reactions and find the magnitude and nature of forces in the members in the lighting gantry structure shown below.



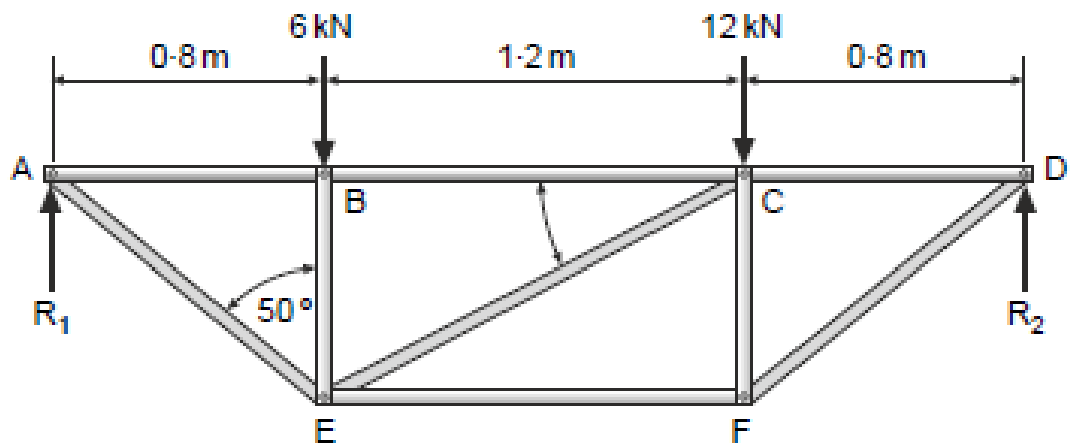
Frame Structures

Task 1

The diagram below shows the structure used to support the lift motor and cable drum.



The free-body diagram below shows the forces acting on one truss.

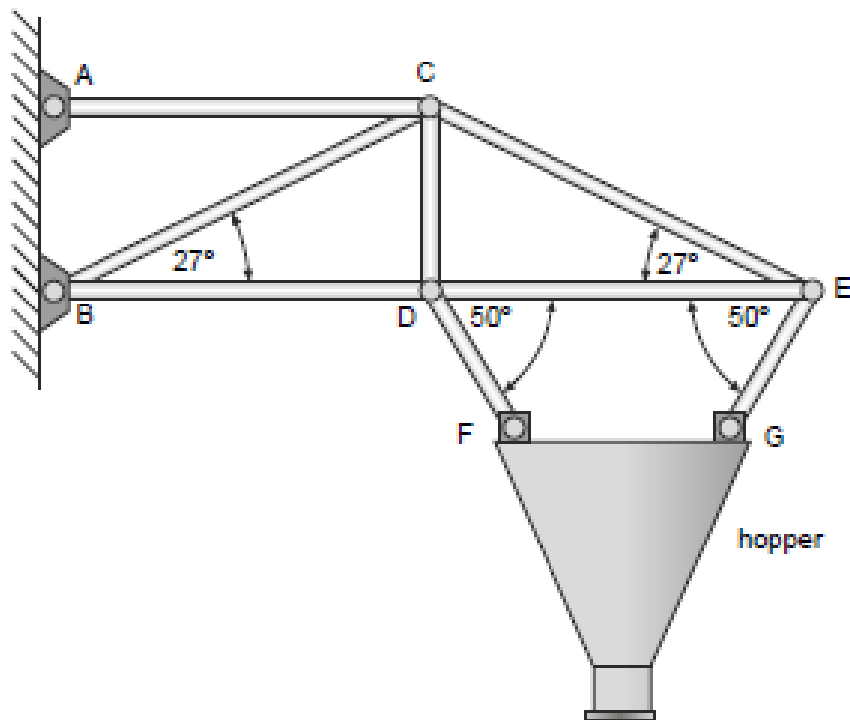


For the loading conditions shown above:

- (a) determine the magnitude of R_1 ;
- (b) use *nodal analysis* to calculate the magnitude and nature of the forces

Task 2

The diagram shows a structure supporting a hopper used for filling lorries with concrete.

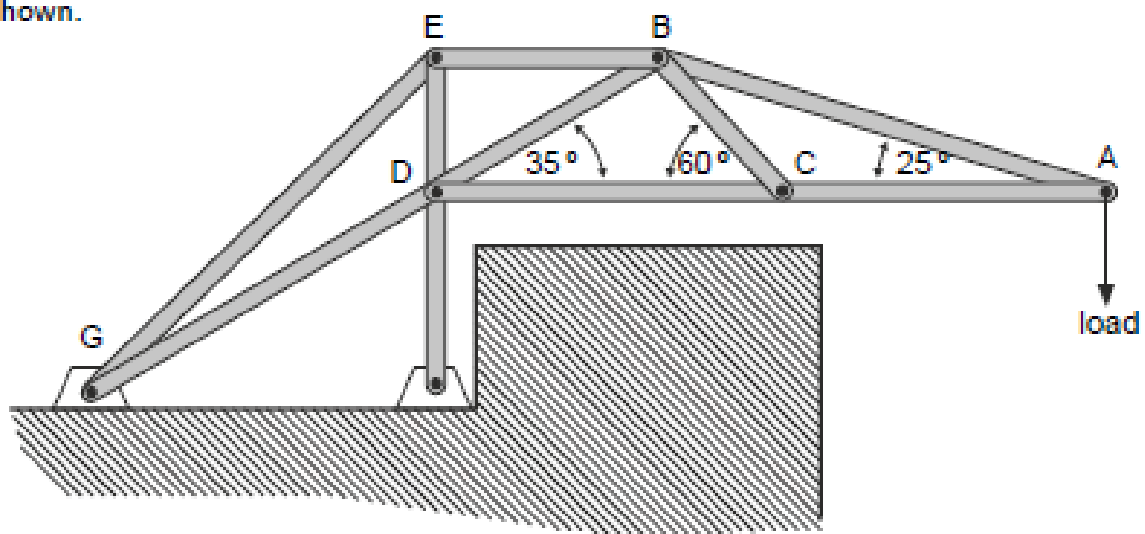


For a particular load, members DF and EG are each subjected to a tensile force of 78 kN.

Using *nodal analysis*, calculate the magnitude and nature of the forces in members CE, DE, CD, BD and BC.

Task 3

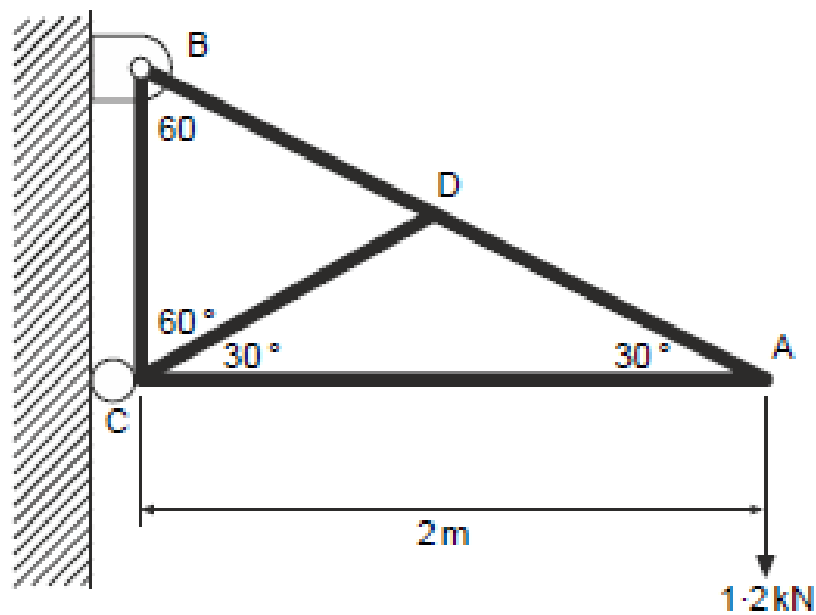
A frame for supporting a maintenance platform suspended from the top of a building is shown.



Calculate, using *Nodal Analysis*, the magnitude and nature of the forces in members AB, AC, BC, CD and BD, for a load of 5 kN acting on the support frame.

Task 4

A frame structure is shown in below.



(a) Calculate the magnitude of:

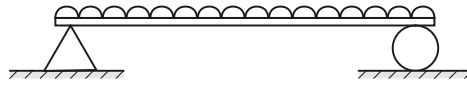
- the reaction force at C, using the Principle of Moments;
- the reaction force at B.

CD is a redundant member.

(b) Explain what is meant by the term "redundant member".

(c) Calculate, using *nodal analysis*, the magnitude and nature of the forces in members AD and AC.

Uniformly distributed load



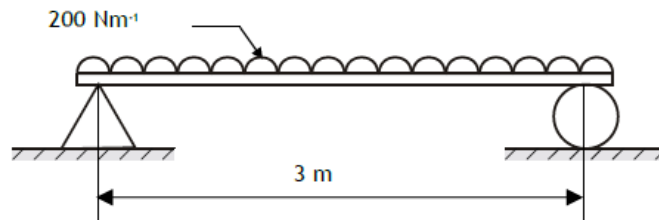
A uniformly distributed load (UDL) is a load which is spread constantly along the length of a beam. In practice, this is the usual type of load a beam will require to support.

The load is given in terms of the total force acting on each metre length of the beam i.e. kN m^{-1} .

In order to calculate the forces acting on the beam, it is assumed that the UDL acts centrally on the beam as a single force.

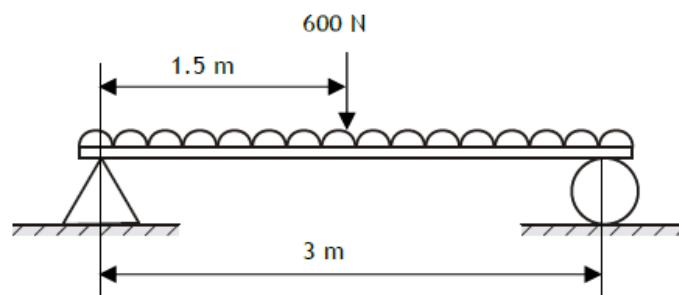
Worked Example

Calculate the force acting on the beam.



$200 \times 3 = 600 \text{ N}$ acting in the centre of the beam (i.e. 1.5 m from the end).

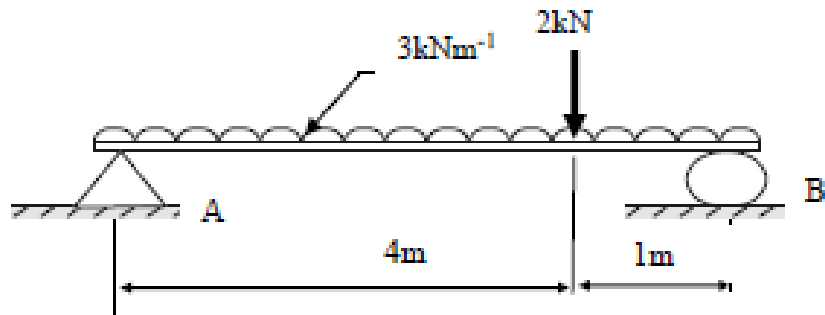
The diagram can be redrawn as shown below.



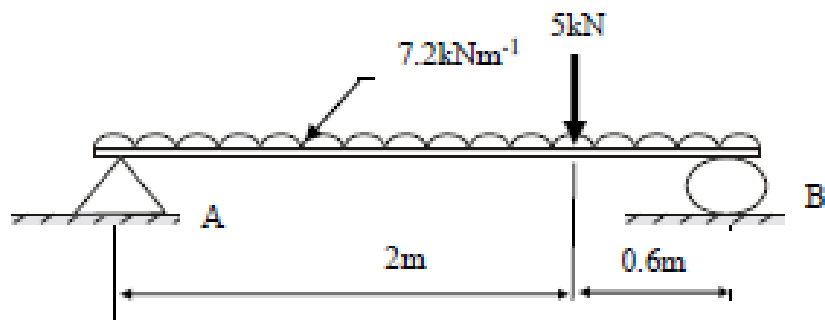
UDLs

Calculate R_A and R_B for each of the questions below.

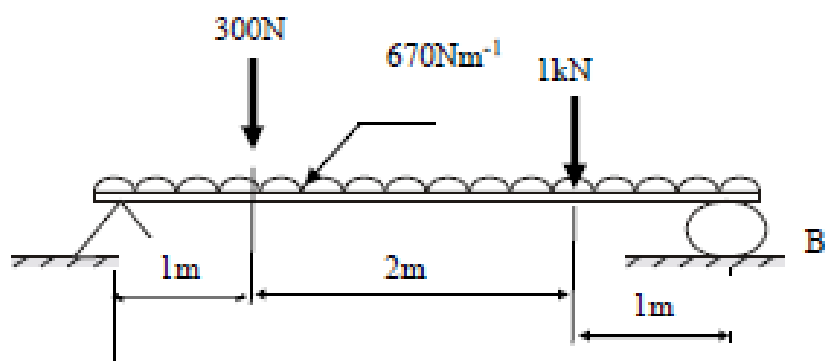
a)



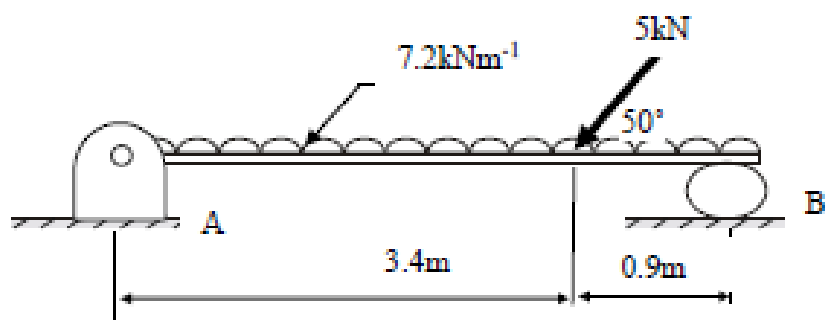
b)



c)



d)



Simultaneous Equations

Sometimes it is possible to solve a frame structure using other mathematical techniques, such as simultaneous equations.

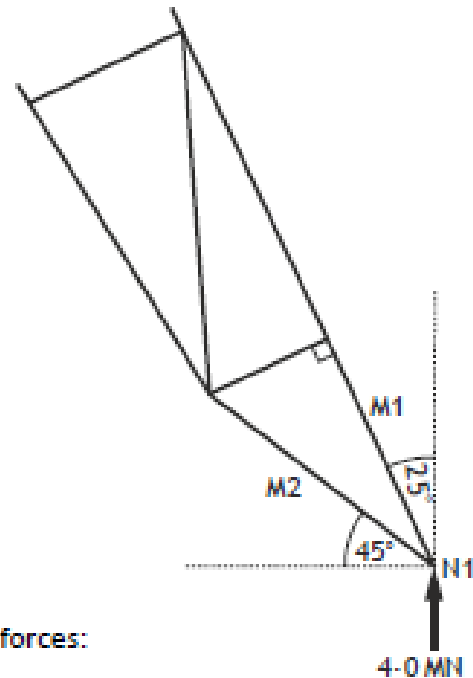
Worked Example

A new railway station is being built. A structural engineering company has been asked to produce detailed information about a possible station roof.

Detail from a proposed design for one of the roof's steel support legs is shown.

Node N1 is in static equilibrium. M2 is a tie.

Calculate the magnitude of the forces in members M1 and M2.



Step 1 - Establish an equation for the sum of the horizontal forces:

$$\begin{aligned}\Sigma F_H &= 0, \\ \Rightarrow M2_H - M1_H &= 0 \\ \Rightarrow M2 \cos(45^\circ) - M1 \sin(25^\circ) &= 0 \\ \Rightarrow 0.707 M2 - 0.4225 M1 &= 0 \text{ [equation 1]}\end{aligned}$$

Step 2 - Establish an equation for the sum of vertical forces:

$$\begin{aligned}\Sigma F_V &= 0 \\ \Rightarrow -M2_V + M1_V - 4.0 &= 0 \\ \Rightarrow -M2 \sin(45^\circ) + M1 \cos(25^\circ) - 4.0 \times 10^6 &= 0 \\ \Rightarrow -0.707 M2 + 0.906 M1 - 4.0 \times 10^6 &= 0 \text{ [equation 2]}\end{aligned}$$

Step 3 - Find a way to combine the two equations. In this case, adding them cancels out the M2 expressions:

$$\begin{aligned}0.906 M1 - 0.4225 M1 - 4.0 \times 10^6 &= 0 \text{ [equation 1 + equation 2]} \\ \Rightarrow 0.484 M1 &= 4.0 \times 10^6 \\ \Rightarrow M1 &= 8.26 \times 10^6 \text{ N} = 8.3 \text{ MN (to 2 significant figures)}\end{aligned}$$

Step 4 - Substitute the value for M1 into one of the equations to find M2:

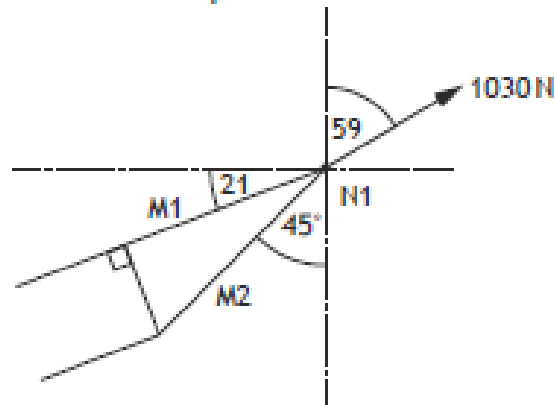
from equation 1,

$$M2 = 0.4225/0.707 \times 8.26 = 4.9 \text{ MN (to 2 significant figures)}$$

Task 1

Part of a fairground ride helps support a car that passengers sit it as it spins around.

As the ride spins, a force of 1030 N acts on a point of the structure as shown in the diagram below.



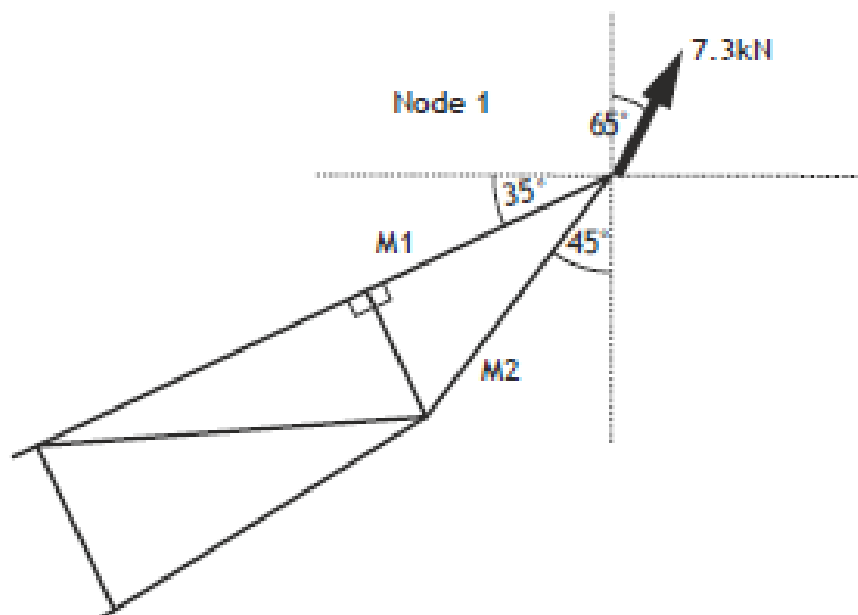
Calculate, using simultaneous equations, the magnitude of the forces in members $M1$ and $M2$.

Task 2

The structure supporting the pod for a centrifugal machine to simulate high forces experienced by astronauts is shown.

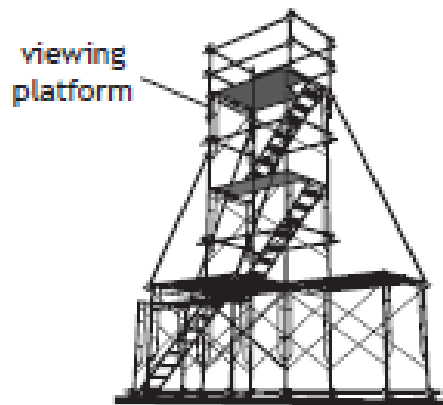
Node $N1$ is in static equilibrium. $M2$ is a tie.

Calculate the magnitude of the forces in members $M1$ and $M2$.

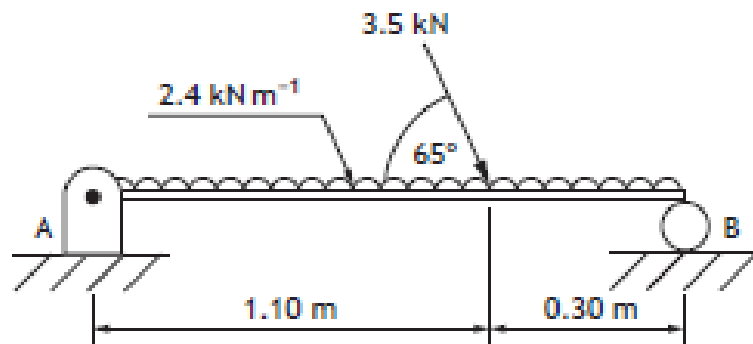


2023 Past Paper

7. Viewing platforms are commonplace in sports academies to allow for filming of training and games.



A beam used in the construction of the viewing platform is shown.



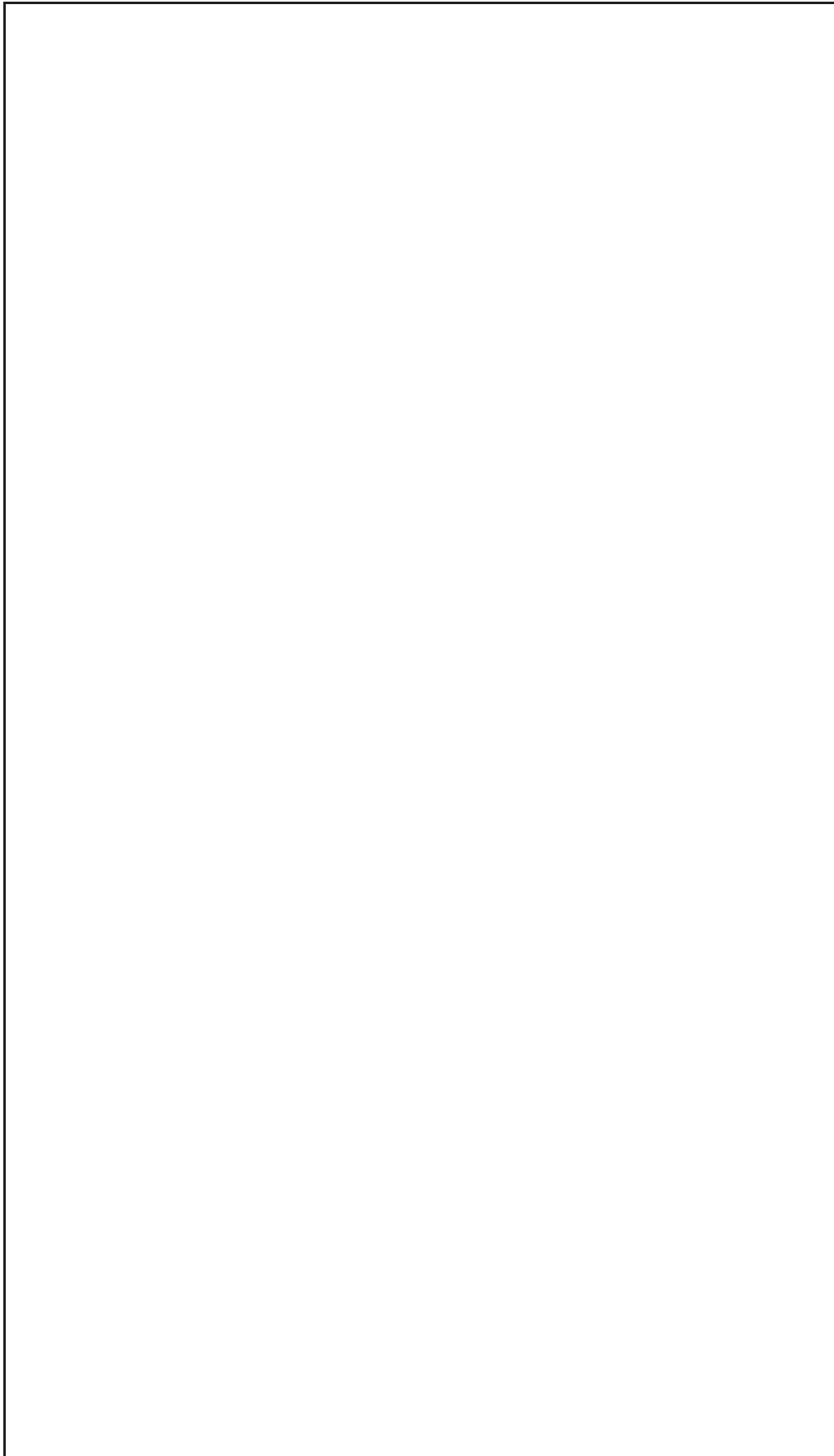
- (a) (i) Calculate the magnitude of the reaction at B.

3

7. (a) (continued)

(ii) Calculate the magnitude and direction of the reaction at A.

4



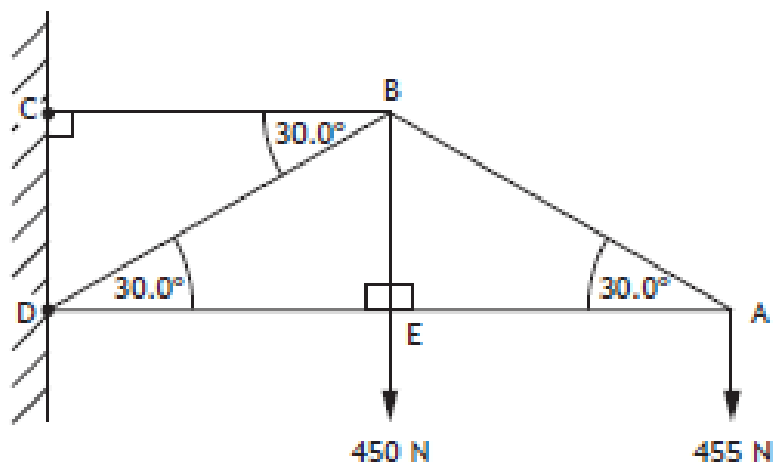
2023 Past Paper

8. (continued)

Due to the rapid expansion of the construction site, additional floodlights are to be installed.



The diagram below shows part of the design for the frame that supports the floodlights.



- (c) Calculate, using nodal analysis at nodes A and B, the magnitude and nature of the forces in members AB, AE, BD, and BC.

Member BE is a 450 N tie.

Complete the table below. Show all working and final units.

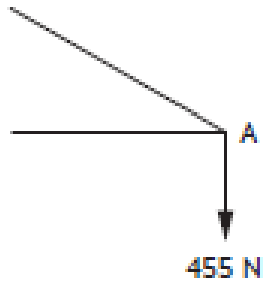
8

Member	Magnitude	Nature
AB		tie
AE		strut
BD		
BC		

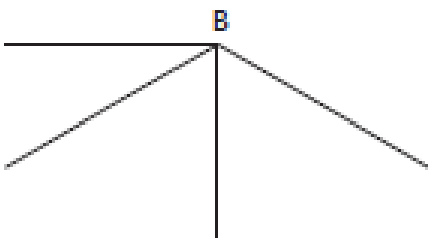
8. (c) (continued)

Space for calculations

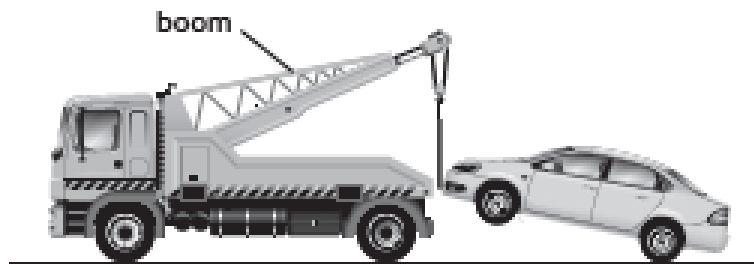
node A



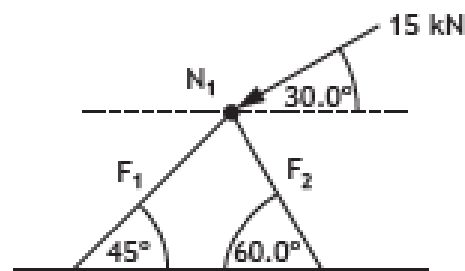
node B



11. (continued)



The structural design for the frame used to support the boom is shown.



Node N_1 is in static equilibrium.

F_1 is a strut, and F_2 is a tie.

- (c) (i) Write, in its simplest form, the equation for the vertical forces acting on N_1 (include all forces and their angles).

1

- (ii) Write, in its simplest form, the equation for the horizontal forces acting on N_1 (include all forces and their angles).

1

11. (c) (continued)

(iii) Calculate the magnitude of the forces in members F_1 and F_2 .

3

